

The hierarchy problem and the self-localized Higgs

C.P. Burgess,^{abc} Claudia de Rham^{ab} and Leo van Nierop^b

^a*Perimeter Institute for Theoretical Physics,
Waterloo ON, N2L 2Y5, Canada*

^b*Department of Physics & Astronomy, McMaster University,
Hamilton ON, L8S 4M1, Canada*

^c*Theory Division, CERN,
CH-1211 Geneva 23, Switzerland*

*E-mail: cburgess@perimeterinstitute.ca, crham@perimeterinstitute.ca,
lvannierop@perimeterinstitute.ca*

ABSTRACT: We examine brane-world scenarios in which all the observed Standard Model particles reside on a brane but the Higgs is an elementary extra-dimensional scalar in the bulk. We show that, for codimension 2 branes, often-neglected interactions between the bulk Higgs and the branes cause two novel effects. First, they cause $\langle H \rangle$ to depend only logarithmically on the UV-sensitive coefficient, m_B^2 , of the mass term, $m_B^2 H^* H$, of the bulk potential, thus providing a new mechanism for tackling the hierarchy problem. Second, the Higgs brane couplings cause the lowest mass KK mode to localize near the brane without any need for geometrical effects like warping. We explore some preliminary implications such models have for the Higgs signature at the LHC, both in the case where the extra dimensions arise at the TeV scale, and in ADD models having Large Extra Dimensions. Novel Higgs features include couplings to fermions which can be different from Standard Model values, m_f/v , despite the fermions acquiring their mass completely from the Higgs expectation value.

KEYWORDS: Field Theories in Higher Dimensions, Higgs Physics, Large Extra Dimensions.

Contents

1. Introduction	1
2. Vacuum energetics of extra-dimensional scalars	3
2.1 Codimension-one	4
2.2 Codimension-two	5
2.3 The self-localized state	8
3. A self-localized bulk Higgs and the hierarchy problem	10
3.1 The model	10
3.2 Scales and naturalness	11
3.3 Higgs-induced mass terms	13
3.4 Couplings to the Higgs fluctuations	15
4. Possible signatures of a bulk Higgs scenario	16
4.1 Inclusive processes	16
4.2 Higgs decays to fermions	18
4.3 TeV-scale compactifications	19
4.4 Large extra dimensions	20
5. Conclusions	21
A. Some properties of Bessel functions	22
B. Classical divergences in brane couplings	23
C. Higher codimension	25
C.1 Coupling renormalization	25
C.2 Boundary condition and energy density	27
C.3 Codimension-3	28
D. Bulk Goldstone modes	28

1. Introduction

In the Standard Model (SM) the Higgs field is in many ways the odd man out. In the absence of the Higgs the only interactions that remain are gauge interactions, characterized by only a handful of coupling constants. But with the Higgs comes the deluge of parameters that parameterize our ignorance of the ultimate origins of the model's many masses and

mixing angles. And among these parameters is the one dimensional quantity, μ , that governs the size of the $\mu^2 H^* H$ term in the Higgs potential, and by fixing the size of the Higgs v.e.v. sets the scale for all masses. It is the sensitivity of this parameter to much heavier scales that is at the root of the hierarchy problem [1].

Historically, the hierarchy problem has been one of the main motivations for exploring brane-world scenarios for physics beyond the Standard Model [2, 3], for which all of the observed SM particles are trapped on a (3+1)-dimensional brane within an extra-dimensional bulk. Motivated by the observations that the Higgs is the lone SM particle yet to be observed, we here explore the idea that it is the only SM particle that is *not* confined to a brane: i.e. whereas all other SM particles live on a brane, the Higgs lives in the bulk. The hope is that this might account for its special role within the SM.

Brane-world models with the Higgs in the bulk have been examined in the literature, most often within the context of 5D Randall-Sundrum constructions [3]. Yet these models differ from the present proposal in one of two ways: either by imagining the extra-dimensional Higgs to be related to other fields by supersymmetry [4, 5]; or by taking the Higgs to be the 4D scalar component of what is ‘really’ an extra-dimensional gauge potential [6, 7]. The motivation for doing so is the expectation that the extra-dimensional gauge symmetries can help alleviate the hierarchy problem, potentially allowing some of the properties of Higgs interactions to be unified with those of the gauge interactions. Implicit in this is the belief that a Higgs that is a bona-fide extra-dimensional scalar makes no progress towards alleviating the hierarchy problems of the usual 4D Higgs.

A model more similar to the one studied here was considered in ref. [8, 9], although from a different point of view. In ref. [8] the authors study the effects of codimension-2 brane couplings on a massless bulk scalar, with a focus on couplings close to the critical value for which the symmetry-breaking properties of the vacuum change. Ref. [9] generalizes to massive bulk fields, but without the focus of this paper on the hierarchy problem, and consequently without the study of couplings to fermions and gauge bosons described herein.

It is simple to see why extra dimensions in themselves are generally believed not to alleviate the hierarchy problem. This is because the Higgs potential,

$$U = -\frac{m_B^2}{2} H^* H + \frac{g}{4} (H^* H)^2, \quad (1.1)$$

is always minimized by $H^* H = m_B^2/g$, where in n dimensions g has the (engineering) dimension of $(\text{mass})^{4-n}$ while m_B always simply has the dimension of mass. But the essence of the hierarchy problem is that because m_B is proportional to a positive power of mass, it generically receives contributions from heavy particles that grow with the mass, M , of the particles involved, and so is dominantly affected by the heaviest such particle that can contribute. Since m_B is a positive power of mass in any number of dimensions it is hard to see how the hierarchy problem can be ameliorated simply by placing the Higgs into the bulk.

In this paper we show why this simple argument is incorrect once the couplings between a bulk Higgs and the brane are properly taken into account. The brane-bulk interactions change the argument because the Higgs potential on the brane, U_b , and in the bulk, U_B , can

disagree on which value of the Higgs v.e.v. has the least energy. In this case the system generically resolves this potential frustration by appropriately balancing these potential energies with the gradient energies which punish the field for attempting to interpolate between the two minima. But *if* the brane has codimension 2 (i.e. there are two dimensions transverse to the brane, such as for a (3+1)-dimensional brane situated in a 6D bulk), the Higgs likes to vary logarithmically near the branes, and the gradient energy associated with this variation is such that the resulting v.e.v. only depends *logarithmically* on the UV-sensitive term, m_B , of the bulk potential. Braneworld models can help with naturalness problems for a number of reasons; brane-bulk couplings provide a new way for them to do so. We show that the lunch is nevertheless not completely free, however, since the hierarchy problem gets partially recast as a requirement for the coefficients of the brane interactions $(H^*H)^2$ and $D_M H^* D^M H$ being required to be suppressed by very different scales. This kind of hierarchical suppression usually does not arise between two operators like these, that are not distinguished by low-energy symmetries or selection rules.

We also show how Higgs-brane interactions change another fundamental piece of widely-held intuition regarding the properties of a bulk Higgs. In the presence of a (positive) extra-dimensional mass term, $U_B = +\frac{1}{2} m_B^2 H^* H$, the spectrum of Kaluza-Klein (KK) states would usually be expected to consist of a multitude of levels (generically spaced by $M_c \sim 2\pi/L$ for a toroidal extra dimension of circumference L , say) that start at energies above a gap, $m_k \geq m_B$. We show here that brane-Higgs interactions can generically introduce a state which lives within this gap, $m < m_B$, that is ‘bound’ in the sense that its wave-function is localized at the position of the brane. We call this the ‘self-localized’ state inasmuch as its localization is a consequence only of the Higgs self-interactions and not on any geometric effects, such as those due to warping.

These arguments are presented in more detail in their simplest context in the next section, §2. §3 then tries to fashion an approach to the hierarchy problem by providing a preliminary discussion of the kinds of interactions that would be required for a realistic model, and the ways in which the low-energy Higgs couplings resemble and differ from those of the SM Higgs, as a function of the scales involved. §4 then follows with a discussion of some of the potential signatures and constraints such a scenario might have for Higgs physics. Our conclusions are briefly summarized in §5.

2. Vacuum energetics of extra-dimensional scalars

In this section we describe the interplay between brane and bulk energetics for the simplest toy model: a single real scalar, ϕ , in the presence of both brane and bulk potentials, U_b and U_B . We first review the more familiar situation of a codimension-1 brane in a 5D bulk, and then contrast this with the codimension-2 case with 6 bulk dimensions. (The situation for higher codimension is sketched in appendix C.) Because they are peripheral to our main point we neglect gravitational effects in what follows, and so assume the mass scales involved are low enough for this to represent a good approximation.

2.1 Codimension-one

We first consider the codimension-one case, reproducing the results of ref. [10]. Consider the following 5D scalar field theory, having both bulk- and brane-localized interactions,

$$S = - \int d^4x dy \left[\frac{1}{2} (\partial_M \phi \partial^M \phi) + U_B(\phi) + \delta(y) U_b(\phi) \right], \quad (2.1)$$

with $\{x^M\} = \{x^\mu, y\}$. The field equation for this model is

$$\partial^M \partial_M \phi - U'_B(\phi) = \delta(y) U'_b(\phi), \quad (2.2)$$

and the integration of this equation across the brane position (assuming continuity of ϕ) further implies the scalar jump condition

$$[\partial_y \phi]_0 = U'_b(\phi_0), \quad (2.3)$$

where $\phi_0 = \phi(y=0)$ and $[A]_0 = A(y=0^+) - A(y=0^-)$. The classical energy density per unit brane volume associated with a given field configuration in this model is then

$$\mathcal{H} = \int_{y_{\min}}^{y_{\max}} dy \left[\frac{1}{2} (\dot{\phi}^2 + (\nabla \phi)^2 + (\partial_y \phi)^2) + U_B(\phi) \right] + U_b(\phi_0), \quad (2.4)$$

where $y_{\min} < 0 < y_{\max}$ and ∇ denotes differentiation in the in-brane spatial directions, $\{x^i\}$.

We now specialize to the case where the field has only a mass term in the bulk, while it has a quartic interaction on the brane. Keeping in mind that ϕ has dimension $(\text{mass})^{3/2}$ in 5 dimensions,

$$U_B(\phi) = \frac{1}{2} m_B^2 \phi^2 \quad \text{and} \quad U_b(\phi) = -\frac{1}{2} m_b \phi^2 + \frac{1}{4M_b^2} \phi^4, \quad (2.5)$$

where $m_b > 0$ is chosen to ensure that the minimum of the brane potential occurs at the nonzero value $\phi^2 = M_b^2 m_b$, in contrast with the bulk potential which is minimized at $\phi = 0$.

Since U_b and U_B are not minimized by the same configuration, the vacuum solution need not correspond to a constant field configuration, $\partial_M \phi = 0$. Since the solutions to the field equations that only depend on y are exponentials, $\phi \propto e^{\pm m_B y}$, the general bulk solution is a linear combination of such terms. If the extra dimension is sufficiently large — $|m_B y_{\min}| \gg 1$ and $m_B y_{\max} \gg 1$ — then we can drop the solutions which grow exponentially far from the brane, just as if the extra dimension were noncompact. In this case the vacuum configuration should vanish at infinity, and the solution is therefore given by

$$\phi(y) = \bar{\phi} e^{-m_B |y|}, \quad (2.6)$$

where $\bar{\phi}$ is to be fixed using the boundary condition, eq. (2.3), at $y = 0$: *i.e.* $-2m_B \bar{\phi} = U'_b(\bar{\phi})$, or

$$\left(2m_B - m_b + \frac{\bar{\phi}^2}{M_b^2} \right) \bar{\phi} = 0. \quad (2.7)$$

When $m_b < 2m_B$ the only real solution allowed is $\bar{\phi} = 0$, but when $m_b > 2m_B$ there are three solutions for $\bar{\phi}$, corresponding to $\bar{\phi} = 0$ and $\bar{\phi} = \pm\phi_c$, with

$$\phi_c^2 = M_b^2 (m_b - 2m_B). \quad (2.8)$$

Since $\mathcal{H} = 0$ for $\bar{\phi} = 0$ and $\mathcal{H} = -\frac{1}{4}M_b^2 (m_b - 2m_B)^2$ for $\bar{\phi}^2 = \phi_c^2$, we see that it is the nontrivial configuration which represents the classical ground state when $m_b > 2m_B$. This can also be seen more generally by writing the energy density as a function of $\bar{\phi}$,

$$\mathcal{H}(\bar{\phi}) = -\frac{1}{2} (m_b - 2m_B) \bar{\phi}^2 + \frac{1}{4M_b^2} \bar{\phi}^4, \quad (2.9)$$

which is indeed minimized, for $m_b > 2m_B$, by $\bar{\phi} = \pm M_b \sqrt{m_b - 2m_B}$, with the unstable stationary point, $\bar{\phi} = 0$, situated at a local maximum.

The resulting vacuum

$$\phi^2(y) = M_b^2 (m_b - 2m_B) e^{-2m_B|y|}, \quad (2.10)$$

extrapolates from the bulk minimum ($\phi = 0$) for large y to the value $\phi_0 = \pm M_b \sqrt{m_b - 2m_B}$ at the brane. This represents a compromise between the bulk minimum, the value $\bar{\phi} = \pm M_b \sqrt{m_b}$, which minimizes U_b , and the gradient energy required to interpolate between the two. Notice that ϕ_0 approaches the brane minimum in the limit where the bulk potential is very flat, $m_B \ll m_b$.

2.2 Codimension-two

We now contrast the previous results with a similar analysis for the codimension-2 case of a real scalar field coupled to a 3-brane in 6 spacetime dimensions, where we show that the larger gradient energy more strongly favors the minimum of the bulk potential relative to that of the brane. Using the action

$$S = - \int d^4x d^2y \left[\frac{1}{2} (\partial_M \phi \partial^M \phi) + U_B(\phi) + \delta^2(y) U_b(\phi) \right], \quad (2.11)$$

we have the equation of motion

$$\partial^M \partial_M \phi - U'_B(\phi) = \delta^2(y) U'_b(\phi). \quad (2.12)$$

Assuming a flat space-time metric

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + r^2 d\theta^2, \quad (2.13)$$

and integration of the equation of motion across a very small disc centered on the brane position at $r = 0$ (assuming continuity of ϕ) further implies the condition

$$\lim_{r \rightarrow 0} [2\pi r \partial_r \phi] = U'_b(\phi_0), \quad (2.14)$$

where r measures the radial distance from the brane situated at $r = 0$. For configurations depending only on r , this corresponds to using the radial field equation

$$\frac{1}{r} \partial_r (r \partial_r \phi) - U'_B(\phi) = \frac{\delta_+(r)}{2\pi r} U'_b(\phi), \quad (2.15)$$

where $\delta_+(r)$ is normalized so that $\int_0^a dr \delta_+(r) f(r) = f(0)$, for any $a > 0$.

Since our interest is in how the system resolves the frustration of minimizing brane and bulk potentials having different minima, we specialize to the simple choices

$$U_B(\phi) = \frac{1}{2} m_B^2 \phi^2 \quad \text{and} \quad U_b(\phi) = -\frac{1}{2} \lambda_2 \phi^2 + \frac{1}{4} \lambda_4 \phi^4, \quad (2.16)$$

with both λ_2 and λ_4 taken to be positive. Keeping in mind a 6D scalar field has dimension (mass)², we see that the parameter λ_2 is dimensionless, while $\lambda_4 = 1/M_b^4$.

Provided the extra dimensional radius, L , satisfies $m_B L \gg 1$, it is a good approximation to demand the bulk vacuum configuration to vanish at large r , leading to the following solution

$$\phi(r) = \bar{\phi} K_0(m_B r), \quad (2.17)$$

where the modified Bessel function, $K_0(z)$, falls exponentially with z for large z and diverges logarithmically as z approaches zero. Using $K_0(z) = -\ln(z/2) - \gamma + O(z)$ to evaluate $r \partial_r \phi \rightarrow -\bar{\phi}$ as $r \rightarrow 0$, allows the boundary condition, eq. (2.14), to be written

$$-2\pi \bar{\phi} = U_b'(\phi_0), \quad (2.18)$$

and here we encounter the first difference from the codimension-2 case: $\phi(r)$ diverges logarithmically as $r \rightarrow 0$, making $\phi_0 = \phi(r=0)$ ill defined. Regularizing¹ by evaluating at a small but nonzero radius, $r = \epsilon$, gives $\phi_\epsilon = \bar{\phi} z_\epsilon$, where

$$z_\epsilon \equiv K_0(m_B \epsilon) = \ell + \ln 2 - \gamma + \mathcal{O}(\epsilon), \quad (2.19)$$

with $\ell = -\ln(m_B \epsilon)$ diverging logarithmically when $\epsilon \rightarrow 0$ and $\gamma = 0.5772\dots$ being the Euler-Mascheroni constant.

The trouble here lies in the fact that the classical solution for the bulk field coupled to a brane diverges when evaluated at the brane source. This is a completely generic feature for branes having codimension 3 or larger — e.g. the divergence of the Coulomb field at the position of the source charge. It is also generic for codimension 2, although exceptions in this case also arise, such as for the conical singularities arising in the static gravitational fields sourced by some [13–15] but not all [16, 11] codimension-2 branes. And the generic resolution to this problem lies in the need to renormalize the brane-bulk couplings *even at the classical level* [17, 18]. As these references show (and is briefly summarized in appendix B), the requirement that bulk ϕ propagators be finite implies the brane couplings also diverge logarithmically in the limit $\epsilon \rightarrow 0$, with the result

$$\lambda_2 = \frac{\bar{\lambda}_2}{1 + \bar{\lambda}_2 \hat{\ell} / 2\pi} \quad \text{and} \quad \lambda_4 = \frac{\bar{\lambda}_4}{\left(1 + \bar{\lambda}_2 \hat{\ell} / 2\pi\right)^4}, \quad (2.20)$$

¹This regularization can be done more precisely by modelling the codimension-2 brane by a small codimension-1 circle at radius $r = \epsilon$, and using the codimension-1 jump conditions to relate the exterior bulk fields to the nonsingular fields in the circle's interior [11, 12].

where the $\bar{\lambda}_i$ are renormalized quantities that remain finite in the limit that $\epsilon \rightarrow 0$, and

$$\hat{\ell} = -\ln(\mu\epsilon) = \ell + \ln\left(\frac{m_B}{\mu}\right), \quad (2.21)$$

for an arbitrary renormalization scale μ . For later purposes we remark that because the term in the action involving λ_2 is quadratic in ϕ , it is possible to evaluate the classical scalar propagator, including the brane-bulk mixing, without having to assume that λ_2 or $\bar{\lambda}_2$ are small (see appendix B for details). In particular the domain of validity of eq. (2.20) includes the regime of large $\bar{\lambda}_2$.

If we regularize by replacing ϕ_0 with ϕ_ϵ , the boundary condition which determines $\bar{\phi}$ becomes

$$-2\pi r \partial_r \phi + U'_b(\phi) = 2\pi \bar{\phi} + U'_b(\phi_\epsilon) = (2\pi - \lambda_2 z_\epsilon + \lambda_4 z_\epsilon^3 \bar{\phi}^2) \bar{\phi} = 0. \quad (2.22)$$

For $\lambda_2 < 2\pi/z_\epsilon$ this only admits the trivial solution, $\bar{\phi} = 0$, but for $\lambda_2 > 2\pi/z_\epsilon$ three solutions are possible: $\bar{\phi} = 0$ and $\bar{\phi} = \pm\phi_c$, with

$$\phi_c^2 = \frac{(\lambda_2 - 2\pi/z_\epsilon)}{\lambda_4 z_\epsilon^2}. \quad (2.23)$$

Notice that the criterion distinguishing the existence of one or three solutions depends only logarithmically on m_B (through its appearance in z_ϵ), and can be phrased in a regularization-independent manner by trading λ_2 for $\bar{\lambda}_2$. In particular, the condition $\lambda_2 < 2\pi/z_\epsilon$ ensuring only $\bar{\phi} = 0$ is a solution then becomes $\bar{\lambda}_2 < 2\pi/c$, where $c = \ln 2 - \gamma - \ln(m_B/\mu)$ defines the finite part of $z_\epsilon \equiv \hat{\ell} + c$.

The physical content of these expressions becomes clearer once the relative energy of these solutions is computed using the classical energy density, $\mathcal{H}(\bar{\phi})$, which is finite once it is expressed in terms of the renormalized quantities $\bar{\lambda}_i$. Explicitly, we have

$$\mathcal{H} = \lim_{\epsilon \rightarrow 0} \left\{ 2\pi \int_\epsilon^\infty r dr \left[\frac{1}{2} (\partial_r \phi)^2 + \frac{1}{2} m_B^2 \phi^2 \right] + U_b(\phi_\epsilon) \right\} \quad (2.24)$$

$$= \lim_{\epsilon \rightarrow 0} \left\{ \pi \bar{\phi}^2 \int_{m_B \epsilon}^\infty dz z \left[(K'_0)^2 + (K_0)^2 \right] + U_b(\phi_\epsilon) \right\}. \quad (2.25)$$

The integral may be evaluated in closed form (see appendix A), to give

$$\begin{aligned} \mathcal{H} &= \lim_{\epsilon \rightarrow 0} \left\{ -\frac{\pi}{2} \bar{\phi}^2 m_B^2 \epsilon^2 K_0(m_B \epsilon) \left[K_0(m_B \epsilon) - K_2(m_B \epsilon) \right] - \frac{1}{2} \lambda_2 \phi_\epsilon^2 + \frac{1}{4} \lambda_4 \phi_\epsilon^4 \right\} \\ &= \lim_{\epsilon \rightarrow 0} \left\{ \pi \bar{\phi}^2 z_\epsilon - \frac{1}{2} \lambda_2 \bar{\phi}^2 z_\epsilon^2 + \frac{1}{4} \lambda_4 \bar{\phi}^4 z_\epsilon^4 + \mathcal{O}(\epsilon) \right\}, \end{aligned} \quad (2.26)$$

which uses the asymptotic form $K_2(m_B \epsilon) \simeq 2/(m_B \epsilon)^2$ for small ϵ . Using the asymptotic limit of eq. (2.20) for $\bar{\lambda}_2 \hat{\ell} \gg 2\pi$,

$$\lambda_2 \simeq \frac{2\pi}{\hat{\ell}} \left[1 - \left(\frac{2\pi}{\bar{\lambda}_2 \hat{\ell}} \right) + \dots \right] \quad \text{and} \quad \lambda_4 \simeq \left(\frac{2\pi}{\bar{\lambda}_2 \hat{\ell}} \right)^4 \bar{\lambda}_4 + \dots, \quad (2.27)$$

we find the finite limit

$$\mathcal{H} = \frac{1}{2} g_2 \bar{\phi}^2 + \frac{1}{4} g_4 \bar{\phi}^4 \quad \text{with} \quad g_2 = 2\pi \left(\frac{2\pi}{\bar{\lambda}_2} - c \right) \quad \text{and} \quad g_4 = \left(\frac{2\pi}{\bar{\lambda}_2} \right)^4 \bar{\lambda}_4, \quad (2.28)$$

where $c = \ln 2 - \gamma - \ln(m_B/\mu)$, as above.

Notice the kinetic energy has combined with the bulk potential energy to partially cancel the quadratic term in the brane potential, with the solution $\bar{\phi} = 0$ being energetically preferred for $\bar{\lambda}_2 < 2\pi/c$ — the same criterion found earlier. Notice also that $c > 0$ if $\mu > \mu_\star = \frac{1}{2} e^\gamma m_B \simeq 0.89 m_B$, and $c < 0$ if $\mu < \mu_\star$. c vanishes at the dividing case, $\mu = \mu_\star$, at which point the quadratic term is simply

$$g_2 = \frac{4\pi^2}{\lambda_{2\star}}, \quad (2.29)$$

with $\lambda_{2\star} \equiv \bar{\lambda}_2(\mu_\star)$. In terms of renormalized quantities the criterion for symmetry breaking becomes $\lambda_{2\star} < 0$, in which case the scalar v.e.v. is

$$\phi_c^2 = -\frac{g_2}{g_4} = -\frac{\lambda_{2\star}^3}{4\pi^2 \bar{\lambda}_4}. \quad (2.30)$$

These calculations illustrate how the vacuum energetics of a bulk scalar depends crucially on the codimension of the brane to which it is coupled. In all cases the competition between gradient and potential energies in general allows the brane potential to drag the bulk scalar v.e.v. away from the value which minimizes U_B . But in the codimension-1 case the marginal strength of brane instability which distinguishes a nonzero from a vanishing v.e.v., $m_b = 2m_B$, depends strongly on the UV-sensitive scale m_B . By contrast, the corresponding criterion for codimension-2 branes, $\bar{\lambda}_2 = 2\pi/c$, is comparatively insensitive to m_B because it is the larger gradient energies which replace U_B in dominating the fight against U_b . (The situation for higher codimension is explored in appendix C, below.)

2.3 The self-localized state

Since we expect the quadratic term in \mathcal{H} to describe the mass of small fluctuations about the background configuration, there is a potential puzzle hidden in the weak dependence of g_2 on m_B . To see why, suppose the two extra dimensions are a square torus of volume $V_2 = L^2$, for which in the absence of the brane interactions we would normally expect a Kaluza Klein spectrum to be labelled by two integers, n_1 and n_2 , with masses

$$M_{n_1 n_2}^2 = m_B^2 + M_c^2(n_1^2 + n_2^2) \geq m_B^2, \quad (2.31)$$

where $M_c = 2\pi/L$. The puzzle is that all of these states have masses larger than m_B , a result which seems hard to reconcile with a mass governed by the size of the quadratic term, $\frac{1}{2} g_2 \bar{\phi}^2$, of \mathcal{H} .

We next show that the resolution of this puzzle lies in the existence of a lower-mass ‘bound’ state whose mass lies in the gap, $m < m_B$, and whose presence relies on the influence of the interactions between ϕ and the brane. Furthermore, this state is localized

near the brane by these interactions, in the sense that its wave-function falls exponentially away from the brane, with a characteristic size of order $a_B \sim 1/k$, where $k^2 = m_B^2 - m^2$. We call this the self-localized state, inasmuch as its localization is a direct consequence of the scalar-brane interactions (rather than due to a geometric effect, like warping, such as considered in ref. [19]).

The fluctuation spectrum. To this end consider small fluctuations in the bulk scalar field,

$$\phi(t, r, \theta) = \varphi(r) + \Phi_{n\omega}(r)e^{in\theta - i\omega t}, \quad (2.32)$$

labelled by their energy, ω , and angular momentum,² n . $\varphi(r)$ here denotes any of the vacuum configurations described above. The field equation obtained by linearizing eq. (2.12) in polar coordinates is

$$\frac{1}{r} \partial_r (r \partial_r \Phi_{n\omega}) - \frac{n^2}{r^2} \Phi_{n\omega} - k^2 \Phi_{n\omega} = \frac{\delta_+(r)}{2\pi r} (-\lambda_2 + 3\lambda_4 \varphi^2) \Phi_{n\omega}, \quad (2.33)$$

where $k^2 = m_B^2 - \omega^2$. For the purposes of identifying the bound state we further specialize to axially symmetric modes, and so set $n = 0$.

The steps for solving for Φ_ω closely parallel those taken above to find the background solution. Away from $r = 0$ the bulk solution is a linear combination of the modified Bessel functions, $K_0(kr)$ and $I_0(kr)$, although in the limit $kL \gg 1$ the admixture of $I_0(kr)$ can be made negligibly small. In this case the background configuration is $\varphi = \bar{\phi} K_0(m_B r)$ and the fluctuation solutions are well approximated by³

$$\Phi_\omega(r) = N_\omega K_0(kr), \quad (2.34)$$

with N_ω an appropriate normalization constant (e.g. $N_\omega^2 = k^2/\pi$ when $kL \gg 1$). (In this notation the tower of KK states having masses greater than m_B correspond to the ordinary Bessel functions obtained when k is pure imaginary.) The eigenvalue, k , is obtained by imposing the boundary condition at $r = 0$, which becomes

$$2\pi N_\omega + U_b''(\varphi) \Phi_\omega(r=0) = (2\pi - \lambda_2 \hat{z}_\epsilon + 3\lambda_4 z_\epsilon^2 \hat{z}_\epsilon \bar{\phi}^2) N_\omega = 0, \quad (2.35)$$

where $z_\epsilon = -\ln(m_B \epsilon/2) - \gamma = \hat{\ell} + c$ is as defined above, and \hat{z}_ϵ is the same quantity with $m_B \rightarrow k$: i.e. $\hat{z}_\epsilon = z_\epsilon + \ln(m_B/k)$. This equation is to be read as being solved for k , leading to the result $\hat{z}_\epsilon = 2\pi/(\lambda_2 - 3\lambda_4 z_\epsilon^2 \bar{\phi}^2)$, or

$$\begin{aligned} \ln\left(\frac{k}{m_B}\right) &= z_\epsilon - \frac{2\pi}{\lambda_2 - 3\lambda_4 z_\epsilon^2 \bar{\phi}^2} = \hat{\ell} + c - \frac{2\pi}{\lambda_2 - 3\lambda_4 (\hat{\ell} + c)^2 \bar{\phi}^2} \\ &= c - (2\pi/\bar{\lambda}_2) - (3\bar{\lambda}_4 \bar{\phi}^2/\bar{\lambda}_2)(2\pi/\bar{\lambda}_2)^3 + \mathcal{O}(1/\hat{\ell}) \\ &\rightarrow -\left(\frac{2\pi}{\lambda_{2*}}\right) \left[1 + \left(\frac{12\pi^2 \bar{\lambda}_4 \bar{\phi}^2}{\lambda_{2*}^3}\right)\right] \quad \text{as } \epsilon \rightarrow 0. \end{aligned} \quad (2.36)$$

²We assume here an axially-symmetric bulk, such as might be generated (say) by two branes.

³Intriguingly, recasting the field equation to remove the single-derivative term, through the redefinition $\phi = \psi/r^{1/2}$, leads to the Schrödinger equation for motion of a point particle in a $1/r^2$ potential supplemented by a δ -function at the origin. This much-studied equation is known to exhibit the interesting phenomena of dimensional transmutation [20] and nontrivial limit cycles [21].

Consequently, $k = m_B e^{-2\pi/\lambda_{2\text{eff}}}$, or

$$\omega^2 = m_B^2 - k^2 = m_B^2 \left[1 - e^{-4\pi/\lambda_{2\text{eff}}} \right], \quad (2.37)$$

where

$$\frac{1}{\lambda_{2\text{eff}}} = \frac{1}{\lambda_{2\star}} \left[1 + \left(\frac{12\pi^2 \bar{\lambda}_4 \bar{\phi}^2}{\lambda_{2\star}^3} \right) \right]. \quad (2.38)$$

Clearly this state lives in the gap, with $\omega < m_B$, provided only that $\lambda_{2\text{eff}} > 0$, and this mass can be hierarchically small if $\lambda_{2\text{eff}} \gg 4\pi$ (which lies within the domain of validity of the approximations used, as emphasized in appendix B).

There are now two cases to consider. When $\lambda_{2\star} > 0$ we have $\bar{\phi} = 0$ and so $\lambda_{2\text{eff}} = \lambda_{2\star} > 0$, showing that the self-localized state exists. In the limit $\lambda_{2\star} \gg 4\pi$ we find $k \simeq m_B$ and $\omega^2 \simeq 4\pi m_B^2 / \lambda_{2\star} = g_2 m_B^2 / \pi \simeq g_2 N_\omega^2$, in agreement with the result computed from $d^2\mathcal{H}/d\bar{\phi}^2$ (once care is taken to canonically normalize the 4D scalar field). Alternatively, when $\lambda_{2\star} < 0$ we have $\bar{\phi} = \pm\phi_c$, with ϕ_c given by eq. (2.30), and so $\lambda_{2\text{eff}} = -\frac{1}{2}\lambda_{2\star} > 0$. Again a bound state exists whose mass agrees with the result, $-2g_2 N_\omega^2$, obtained by differentiating $\mathcal{H}(\bar{\phi})$.

3. A self-localized bulk Higgs and the hierarchy problem

Because the above vacuum energetics show that the expectation value of a bulk scalar coupled to a codimension-2 (or higher codimension) brane is less sensitive to the details of the model's ultraviolet completion it can be used to provide a new approach to tackling the stability issue of the hierarchy problem. This section builds a simple illustrative example of this mechanism, in order to get a sense of its implications.

3.1 The model

The mechanism's defining assumption is that the usual Standard Model Higgs doublet, $H(x, y)$, is located in an extra-dimensional bulk, while all of the other Standard Model particles — *i.e.* its gauge fields, $A_\mu^a(x)$, and fermions, $\psi_k(x)$ — reside on a brane whose codimension is at least two. (In practice we focus on the codimension-2 case in what follows, but generalizations to more general codimension are conceptually straightforward.) We take the brane potential to prefer an $SU_L(2) \times U_Y(1)$ breaking phase, while the bulk potential favors $SU_L(2) \times U_Y(1)$ invariance:

$$U_B = m_B^2 H^* H \quad \text{and} \quad U_b = -\lambda_2 H^* H + \lambda_4 (H^* H)^2, \quad (3.1)$$

where m_B^2 , λ_2 and λ_4 are all real and positive (evaluated at $m_B \epsilon \ll 1$).

We have seen that the classical vacuum of the higher-dimensional theory depends crucially on the sign of the renormalized coupling, $\lambda_{2\star}$, defined at the (large) scale $\mu_\star \simeq 0.89 m_B$. Notice in this regard that eq. (2.20) implies that both signs of $\lambda_{2\star}$ can be consistent with positive λ_2 when $\ell = -\ln(m_B \epsilon)$ is sufficiently large. We take $\lambda_{2\star} < 0$ in order to ensure that the total classical energy is minimized by an $SU_L(2) \times U_Y(1)$ breaking configuration.

If we had had $SU_L(2) \times U_Y(1)$ invariance throughout the bulk we would at this point be able to perform a gauge transformation to ensure that the Higgs doublet everywhere

takes the unitary gauge form, $H = \frac{1}{\sqrt{2}}(0, \chi)^T$, with χ real. However because we only have gauge invariance at the brane position this choice can only be made at $y^m = 0$: $H_0 = \frac{1}{\sqrt{2}}(0, \chi_0)^T$, where $H_0 \equiv H(x, 0)$. Away from the brane H in general contains 4 real fields, $H = \frac{1}{\sqrt{2}}(\zeta_1 + i\zeta_2, \chi + i\zeta_3)^T$, each of which must solve its appropriate field equations.

The arguments of the previous sections imply that the classical vacuum solutions may be constructed in terms of $K_0(m_B r)$ and $I_0(m_B r)$, with the coefficient of $I_0(m_B r)$ negligibly small when the extra dimensions are large compared with m_B^{-1} — i.e. $m_B L \gg 1$:

$$\zeta_i(r) = \bar{\zeta}_i K_0(m_B r) \quad \text{and} \quad \chi(r) = \bar{\chi} K_0(m_B r). \tag{3.2}$$

As before the normalizations, $\bar{\zeta}_i$ and $\bar{\chi}$, are determined by the boundary conditions at $r = 0$, and so $\bar{\zeta}_i = 0$ due to the choice of unitary gauge at the brane, which implies $\zeta_i(0) = 0$ there. By contrast, the arguments of previous sections go through verbatim to imply $\bar{\chi} \equiv V^2$, with

$$V^4 = -\frac{g_2}{g_4} = -\frac{\lambda_{2*}^3}{(2\pi)^2 \lambda_4} = -\frac{\lambda_{2*}^3}{(2\pi)^3} M_b^4, \tag{3.3}$$

where we define $\bar{\lambda}_4/2\pi = 1/M_b^4$.

Similar arguments for the fluctuations, δH , show that in general all four components, $\delta\zeta_i$ and $\delta\chi$, are nonzero in the bulk. However the choice of unitary gauge at the brane endows $\delta\zeta_i$ with the boundary condition that it must vanish, and this in turn implies that none of these fields localizes at the branes in the same way that $\delta\chi$ does.

Since $SU(2) \times U(1)$ is only a global symmetry in the bulk, one might worry that its breaking by H implies that the $\delta\zeta_i$ contain KK towers of Goldstone modes that are systematically light compared with m_B . These could be phenomenologically dangerous, even if their couplings must be derivatively suppressed [22]. However (as shown in appendix D in more detail) the only Goldstone modes in the bulk-Higgs sector are the three self-localized states for the fields $\delta\zeta_i$ that are eaten by the brane gauge fields *via* the usual Higgs mechanism. All other states with energies smaller than m_B are typically removed by the boundary condition that requires $\delta\zeta_i$ to vanish at the brane, leaving the lightest remaining *bona fide* KK modes in $\delta\zeta_i$ with a mass of order m_B .

3.2 Scales and naturalness

We now ask how V depends on the other scales in the problem, in order to identify whether the choices required to have sufficiently small masses for electroweak gauge bosons are technically natural — i.e. stable against integrating out very heavy degrees of freedom.

The model potentially involves several scales: among which are the compactification scale, M_c ; the scale of extra-dimensional gravity, $M_* \gg M_c$, (or perhaps the string scale), which controls our neglect of gravitational physics; the scale of brane structure,⁴ $\Lambda = 1/\epsilon$, used in earlier regularizations, and so on. In principle the UV scale, $M \gg M_c$, to which we imagine being potentially sensitive, can be any one of these, or some other scale associated with other types of heavy particles.

⁴For instance, such structure might ultimately arise if the 3-brane were really a higher-dimensional brane wrapped about further, smaller extra dimensions.

Our choices of scales are restricted by the domain of validity of approximations used in our calculations. For instance, use of codimension-2 branes without resolving the brane structure when discussing the UV physics implies $\Lambda \gg M$. Ignoring (for simplicity) the influence of the second brane on the mode functions (i.e. dropping the admixture of $I_0(kr)$) assumes $k \gg M_c$, where $k^2 = m_B^2 - m^2$ for the self-localized mode. Neglect (for convenience) of gravitational effects requires both $M_* \gg M_c$, and the condition that the spacetime curvatures generated by the configurations of interest to be small compared with M_*^2 . For instance if H takes values of order V^2 that change over distances of order ϵ , then the resulting gradient energies do not overly gravitate if $(\partial H)^2/M_*^6 \sim (V/M_*)^4(\Lambda/M_*)^2 \ll 1$. In what follows we assume all of these conditions to hold. The question we ask is *not* whether these hierarchies themselves are stable under renormalization (as this would require more information, such as specifying a stabilization mechanism for the size of the extra dimensions), but rather whether the choices required of the Higgs potential to obtain an acceptably small V are stable against renormalization, given the presence of these (and possibly other) scales.

This requires an estimate of the corrections to U_B and U_b that might arise as various kinds of heavy particles are integrated out. Although a precise statement of this requires specifying the theory's UV completion, some generic statements are possible on dimensional grounds for the corrections due to integrating out heavy particles that interact through small dimensionless couplings. This is because if such a particle has a large mass M , then its generic contribution to a coupling, λ_i , having dimension (mass) ^{n} is $\delta\lambda_i \propto M^n$. According to this kind of estimate we expect

$$\delta m_B^2 \propto M^2, \quad \delta\lambda_2 \propto \ln M \quad \text{and} \quad \delta\lambda_4 \propto M^{-4}. \tag{3.4}$$

As a result it is natural to expect the corrections to m_B to be dominated by the heaviest particles that can contribute, and so generically expect m_B to be comparable to the largest scales in the problem (and in particular to satisfy $m_B \gg M_c$ and $m_B \gg M_W$). It is the large size of these contributions to m_B that underlie the usual formulation of the hierarchy problem in 4 dimensions, because in this case the scale of the Higgs v.e.v. turns out to be proportional to $|m_B|$.

By contrast, in the 6D model of present interest we have seen that the size of the Higgs v.e.v. is largely independent of m_B , depending dominantly on the dimensionless coupling $\bar{\lambda}_2$ and the dimensionful coupling $\bar{\lambda}_4$. But $\bar{\lambda}_2$ is dimensionless, and so tends to depend only logarithmically on the large UV scale M . Potentially more dangerous is $\bar{\lambda}_4/2\pi = 1/M_b^4$ since this more directly sets the size of V . However this is also not UV sensitive because corrections to it vary inversely with the relevant particle mass on dimensional grounds, and so are dominated by the contributions of the *lightest* particles, rather than the heaviest.

As stated above, we emphasize that our goal here is not to provide an ultraviolet completion of the bulk-Higgs model, as would be required to understand in detail the conditions necessary to produce a large hierarchy in the first place, as this goes beyond the scope of this paper. Our goal is instead to point out how the introduction of Higgs bulk-brane couplings allows interestingly different mass-dependence in low-energy observables, and to study what this might imply for the low-energy sector.

3.3 Higgs-induced mass terms

The phenomenology of any such Higgs hinges on the form of its couplings to observed Standard Model particles, which are assumed in this framework to be localized on a brane.

Gauge couplings. At first sight it is bizarre to restrict the SM gauge fields to a brane and yet allow a charged matter field (the Higgs doublet) live in the bulk. This is bizarre because the $SU_L(2) \times U_Y(1)$ symmetry transformations are global transformations in the bulk (since there is no spin-one field there to ‘gauge’ them), yet are local on the brane. Nonetheless, it must be possible because we could imagine the UV completion of the brane of interest being an ordinary gauge-Higgs theory containing vortex- or domain-wall-type defects. Since the Higgs field defining the defect typically vanishes at the interior of such a vortex/domain-wall, there generically should be spin-1 states which would be very massive given the nonzero Higgs in the bulk, but which can remain light by being localized on the brane. (D -branes also contain localized spin-1 fields.)

More precisely, it can be shown that gauge invariance of such a theory can always be ensured through an appropriate choice of effective interactions (or counter-terms) on the brane. Slightly generalizing the discussion of ref. [23] to codimension two, we may see this formally by taking the Higgs covariant derivatives to be

$$D_M H(x, y) = \partial_M H(x, y) - \delta^2(y) \delta_M^\mu i \kappa g A_\mu^a(x) T_a H(x, y). \quad (3.5)$$

Here T_a are gauge generators, and as before the x^μ lie along the brane directions while the y^m are transverse. g here denotes the dimensionless gauge coupling on the brane and κ is a dimensionful constant, required in order to counter the dimensions of the delta function. One reason for the need for brane counter-terms can be seen because the off-brane components of this covariant derivative are *not* actually covariant under the gauge transformation

$$\delta H(x, y) = \delta^2(y) i \kappa \Omega^a(x) T_a H(x, y), \quad (3.6)$$

even when supplemented by the standard x^μ -dependent nonabelian transformations of $A_\mu^a(x)$. They are not because there is no gauge potential in $D_m H = \partial_m H$ to cancel the term arising when the derivative acts on the delta function. There is however a counterterm that can be added on the brane such that the entire combination *is* gauge invariant.

The implications of a bulk Higgs v.e.v. for gauge boson masses can be seen by writing out the bulk and brane kinetic terms

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= - \int d^2 y D_M H^* D^M H - \kappa_b \mathcal{D}_\mu H_0^* \mathcal{D}^\mu H_0 \\ &= - \int d^2 y \left[\partial_M H^* \partial^M H \right] - (\kappa + \kappa_b) \mathcal{D}_\mu H_0^* \mathcal{D}^\mu H_0 + \kappa \partial_\mu H_0^* \partial^\mu H_0 \\ &\quad + \frac{\kappa g^2}{2} [1 - \kappa \delta^2(0)] (H_0^* \{T_a, T_b\} H_0) A_\mu^a A^{b\mu}, \end{aligned} \quad (3.7)$$

where $\mathcal{D}_\mu H_0 \equiv \partial_\mu H_0 - i g A_\mu^a T_a H_0$ is the standard covariant derivative on the brane. This shows that all of the gauge-boson mass terms appear in the brane kinetic term provided

$\kappa \delta^2(0) = 1$ (and so $\kappa = O(\epsilon^2)$). Notice that this implies $\kappa + \kappa_b \sim \kappa_b$ for any scale $\kappa_b \gg O(\epsilon^2)$.

Superficially the gauge-boson mass obtained from these equations diverges as $\epsilon \rightarrow 0$, due to the divergence there of H_0 . However, this divergence is countered by the renormalization of all Higgs-brane interactions due to the generic ‘dressing’ of these couplings [17, 18] by the Higgs-brane mixing, λ_2 :

$$\kappa_b = \frac{\bar{\kappa}_b}{(1 + \bar{\lambda}_2 \hat{\ell}/2\pi)^2}. \quad (3.8)$$

Going to unitary gauge at the brane position, for which $H_0 = \frac{1}{\sqrt{2}} (0, \chi_0)^T$, with $\langle \chi \rangle = V^2 K_0(m_{Br})$, we then have $\kappa_b \langle \chi_0 \rangle^2 = \bar{\kappa}_b V^4 (2\pi/\lambda_{2*})^2$ as $\epsilon \rightarrow 0$.

The $SU_L(2) \times U_Y(1)$ doublet structure of the Higgs then leads in the standard way to the prediction $M_Z = M_W / \cos \theta_W$, where θ_W is the weak mixing angle, and the W -boson mass is, $M_W = \frac{1}{2} gv$, with

$$v^2 = \left(\frac{2\pi}{\lambda_{2*}} \right)^2 \bar{\kappa}_b V^4 = (246 \text{ GeV})^2. \quad (3.9)$$

Taking $\bar{\kappa}_b = 1/f^2$, this shows that successful phenomenology requires $V^2 = fv(|\lambda_{2*}|/2\pi)$: i.e. V is the geometric mean between 246 GeV and the scale $|\lambda_{2*}|f/2\pi$:

$$V \sim 10^9 \text{ GeV} \left(\frac{|\lambda_{2*}|f/2\pi}{10^{15} \text{ GeV}} \right)^{1/2}. \quad (3.10)$$

Recall that within the present framework we have $V^4 = |\lambda_{2*}/2\pi|^3 (2\pi/\bar{\lambda}_4)$ — c.f. eq. (3.3) — so defining M_b by $\bar{\lambda}_4/2\pi = 1/M_b^4$ as before we see that eq. (3.9) also implies that M_b must be of order

$$M_b \sim \sqrt{vf} \left(\frac{2\pi}{|\lambda_{2*}|} \right)^{1/4}. \quad (3.11)$$

This requires either $M_b \sim f\sqrt{2\pi/|\lambda_{2*}|} \sim v$, or a hierarchy $v \ll M_b \ll f\sqrt{2\pi/|\lambda_{2*}|}$, if $f\sqrt{2\pi/|\lambda_{2*}|} \gg v$. In the absence of a symmetry which forbids a Higgs kinetic term but allows a quartic $(H^*H)^2$ interaction on the brane, naturalness argues we should take f and M_b to be the same order of magnitude, in which case any hierarchy between M_b and v must be due to $|\lambda_{2*}|/2\pi$ being very large or very small. Furthermore, having f and M_b both larger than v requires $|\lambda_{2*}|/2\pi \lesssim O(1)$.

Fermion couplings. Fermion masses in this picture are similarly given by Yukawa couplings between brane-based fermions, ψ_k , and the bulk Higgs doublet. In unitary gauge on the brane, $H_0 = \frac{1}{\sqrt{2}} (0, \chi_0)^T$, these have the form

$$\mathcal{L}_{\text{yuk}} = \frac{y_{ij}}{F} (\bar{\psi}_i \psi_j) \chi_0, \quad (3.12)$$

for y_{ij} a collection of dimensionless Yukawa couplings, and F representing an appropriate ultraviolet scale. The resulting fermion masses are

$$m_{ij} = \frac{y_{ij}}{F} \langle \chi_0 \rangle = \frac{2\pi \bar{y}_{ij} V^2}{\lambda_{2*} F} \quad (3.13)$$

with

$$y_{ij} = \frac{\bar{y}_{ij}}{1 + \bar{\lambda}_{2\star} \hat{\ell} / (2\pi)}, \quad (3.14)$$

being the renormalized Yukawa coupling, as required to counter the divergence of H at the brane position, and the second equality in eq. (3.13) uses $y_{ij} \langle \chi_0 \rangle = 2\pi \bar{y}_{ij} V^2 / \lambda_{2\star}$ in the limit $\epsilon \rightarrow 0$, where $\langle \chi \rangle = V^2 K_0(m_B r)$.

Since \mathcal{L}_{yuk} breaks flavor symmetries — unlike the Higgs kinetic terms — the scale F need not be of the same order of magnitude as⁵ f . In particular, since the dominant contributions to couplings having dimensions of inverse mass come from the lightest scales to contribute, F is typically set by the smallest UV scale which involves flavor-violating physics while f can be much smaller than this. Because of this eqs. (3.9) and (3.13) may contain the seeds of an explanation of the observed smallness of most fermion masses relative to those of the electroweak gauge bosons, since

$$\frac{m_{ij}}{M_W} = \frac{\bar{y}_{ij}}{g} \left(\frac{2f}{F} \right). \quad (3.15)$$

Even a mild hierarchy, $F \gg f$, removes some of the burden of having to require \bar{y}_{ij}/g to be very small.

3.4 Couplings to the Higgs fluctuations

We have seen that the spectrum of fluctuations in the Higgs field generically contains an assortment of KK modes, many of whose masses start above a large gap, $m_{KK} \gtrsim m_B$. For m_B sufficiently large these modes need not play an important role in low-energy observables. The two exceptions to the above statement are the bulk Goldstone modes, whose masses are generically of order M_c , and the self-localized state whose mass can lie within the gap below m_B , and be hierarchically smaller if $|\lambda_{2\star}| \gg 2\pi$. Furthermore, this latter state is present regardless of whether or not the Higgs v.e.v. is nonzero. These light states are likely to be the ones relevant to Higgs phenomenology in Bulk Higgs models, and so this section computes their couplings.

The bulk Goldstone modes. The simplest couplings to compute are those of the bulk Goldstone modes, $\delta\zeta_i$, because their vanishing at the brane position guarantees they completely drop out of any brane couplings that depend only on H_0 or $\partial_\mu H_0$, and not on off-brane derivatives like $\partial_m H_0$. In particular this ensures their removal (in unitary gauge) from the fermion Yukawa couplings and gauge couplings described above.

The self-localized state. Normalizing the wave-function of the self-localized state in the extra dimensions gives a canonically normalized 4D state h , where $\chi = h(x) N_\omega K_0(kr)$, so $y_{ij} \chi = (2\pi/\lambda_{2\star})(k/\sqrt{\pi}) \bar{y}_{ij} h$, with $k^2 = m_B^2 - m_h^2$. The couplings of h to fermions are then given by interactions of the form

$$\mathcal{L}_{4D} = \frac{2\pi \bar{y}_{ij}}{\lambda_{2\star}} \left(\frac{k}{\sqrt{\pi} F} \right) (\bar{\psi}_i \psi_j) h, \quad (3.16)$$

⁵This could arise, say, if the 3-brane is really a higher-dimensional brane wrapped in extra dimensions, and the flavor structure is associated with this wrapping, since this would suggest $F \simeq \Lambda \gg f$.

leading to dimensionless ‘physical’ Yukawa couplings of order

$$\hat{y}_{ij} = \frac{2\pi\bar{y}_{ij}}{\lambda_{2\star}} \left(\frac{k}{\sqrt{\pi F}} \right) = y_{ij}^{\text{sm}} \left(\frac{m_B}{\sqrt{\pi f}} \right) \left(\frac{2\pi}{\lambda_{2\star}} \right) e^{-4\pi/|\lambda_{2\star}|}, \quad (3.17)$$

where the argument of the exponential assumes $\lambda_{2\star} < 0$ (as required for a nonzero Higgs v.e.v.), and the last equality compares to what would be expected in the SM:

$$y_{ij}^{\text{sm}} \equiv \frac{m_{ij}}{v} = \bar{y}_{ij} \left(\frac{f}{F} \right). \quad (3.18)$$

Notice that the quantity $(2\pi/|\lambda_{2\star}|) \exp[-4\pi/|\lambda_{2\star}|]$ falls to zero for large and small $|\lambda_{2\star}|$, taking the maximum value of 0.18 when $|\lambda_{2\star}|/2\pi = 2$.

These expressions show that the self-localized Higgs couplings, \hat{y}_{ij} , can differ significantly from what would be expected in the SM, for two reasons. First, \hat{y}_{ij} can be larger than y_{ij}^{sm} if $m_B \gg f$, and if sufficiently large the self-localized state becomes a strongly coupled broad resonance. Second, \hat{y}_{ij} can differ from y_{ij}^{sm} because of its dependence on $\lambda_{2\star}/2\pi$, which acts to suppress $\hat{y}_{ij}/y_{ij}^{\text{sm}}$ in the limit that $|\lambda_{2\star}|/2\pi$ is either very large or very small. This possibility of having \hat{y}_{ij} differ from the SM expectation contrasts with 4D intuition based on the couplings of a single scalar whose v.e.v. generates mass, since such a scalar must have couplings given by the ratio m_{ij}/v . The reason this conclusion does not hold in the extra-dimensional case is that because the v.e.v., $\langle H(x, 0) \rangle$, responsible for generating masses generically receives contributions from many KK modes and not just the v.e.v. of the single 4D self-localized state, h .

4. Possible signatures of a bulk Higgs scenario

We next sketch some of the qualitative signatures and constraints that might be expected for the kind of Higgs scenario described above. What is to be expected depends somewhat on the choices made for the various scales in the problem, so we divide the discussion according to four simple options according to whether or not we take $|\lambda_{2\star}|$ to be large or small, and whether we take $M_c \sim 1 \text{ TeV}$, or $M_c \sim 10^{-2} \text{ eV}$ (as for large-extra-dimensional models).

4.1 Inclusive processes

We first consider inclusive processes for which a specific Higgs state is not measured, and so which involve a summation over all possible KK modes. These are largely insensitive to the specifics of individual modes, such as the details of the self-localized state.

Fermion-fermion scattering. An important inclusive observable is the rate for fermion-fermion scattering mediated by a virtual Higgs. The amplitude for this process is of order

$$\mathcal{A}(\psi_i\psi_j \rightarrow H \rightarrow \psi_r\psi_s) \simeq \frac{y_{ij}y_{rs}}{F^2} iG_p(0;0) \delta^4(p_i + p_j - p_r - p_s), \quad (4.1)$$

where $p^\mu \equiv (p_i + p_j)^\mu = (p_r + p_s)^\mu$. Here $G_p(y; y')$ is the bulk Higgs propagator, Fourier transformed in the brane directions, x^μ , but evaluated in position space in the off-brane

directions, y^m . $G_p(0;0)$ denotes the same quantity evaluated at the brane position, and is given (see appendix B for details) in terms of the corresponding propagator in the absence of brane-Higgs couplings, $D_p(y; y')$, by

$$G_p(0;0) = \frac{D_p(0;0)}{1 - i\lambda_2 D_p(0;0)}. \quad (4.2)$$

Eliminating y_{ij} , y_{rs} and λ_2 in terms of the renormalized quantities, \bar{y}_{ij} , \bar{y}_{rs} and $\bar{\lambda}_2$, and taking $\epsilon \rightarrow 0$, we find the finite result

$$\begin{aligned} \mathcal{A}(\psi_i \psi_j \rightarrow H \rightarrow \psi_r \psi_s) &\simeq \frac{\bar{y}_{ij} \bar{y}_{rs}}{\bar{\lambda}_2 F^2} \left[\frac{1}{1 - i\bar{\lambda}_2 D_p^\mu(0;0)} \right] \delta^4(p_i + p_j - p_r - p_s) \\ &\simeq \frac{y_{ij}^{\text{sm}} y_{rs}^{\text{sm}}}{\bar{\lambda}_2 f^2} \left[\frac{1}{1 - i\bar{\lambda}_2 D_p^\mu(0;0)} \right] \delta^4(p_i + p_j - p_r - p_s), \end{aligned} \quad (4.3)$$

where $iD_p^\mu(0;0) = (1/2\pi) \ln(\mu/P)$, where $P^2 = p^2 + m_B^2$.

If this same process were computed using the exchange of a massive 4D SM Higgs scalar, we'd have instead obtained

$$\mathcal{A}^{\text{sm}}(\psi_i \psi_j \rightarrow H \rightarrow \psi_r \psi_s) \simeq y_{ij}^{\text{sm}} y_{rs}^{\text{sm}} \left[\frac{1}{p^2 + m_H^2} \right] \delta^4(p_i + p_j - p_r - p_s), \quad (4.4)$$

and so the leading effect is to replace the scale $p^2 + m_H^2$ by $\bar{\lambda}_2 f^2 [1 - i\bar{\lambda}_2 D_p^\mu(0;0)]$. The absence of an observed signal therefore implies the order-of-magnitude bound

$$\bar{\lambda}_2 f^2 \left[1 + \frac{\bar{\lambda}_2}{2\pi} \ln \left(\frac{P}{\mu} \right) \right] \gtrsim O(100 \text{ GeV})^2, \quad (4.5)$$

where $P^2 = (p_i + p_j)^2 + m_B^2 = (p_r + p_s)^2 + m_B^2$.

If reactions of this type were to mediate flavor-changing neutral currents, the strong restrictions on these could potentially bound the scale F to be quite large. However, because the Yukawa couplings can have the same flavor structure as in the SM, there can be a GIM mechanism at work [24] that naturally suppresses the dangerous flavor-changing neutral current (FCNC) reactions produced by bulk-Higgs exchange. We henceforth assume this to be true, and therefore do not further worry about bounds on the fermion couplings due to FCNCs.

Vacuum polarization. As is well known, the contributions to loops of the SM Higgs is well constrained by precision electroweak measurements. The main source of these contributions is through the Higgs contribution to the vacuum polarization of the electroweak gauge bosons. For an extra-dimensional bulk Higgs, this contribution is of order

$$\begin{aligned} \Pi_{ab}^{\mu\nu}(p) &\simeq g^2 \kappa_b^2 \text{tr}(T_a T_b) \int \frac{d^4 q}{(2\pi)^4} (2p - q)^\mu (2p - q)^\nu iG_q(0;0) iG_{p-q}(0;0) \\ &\simeq \frac{g^2 \bar{\kappa}_b^2}{\bar{\lambda}_2^2} \text{tr}(T_a T_b) \int \frac{d^4 q}{(2\pi)^4} \left[\frac{(2p - q)^\mu (2p - q)^\nu}{[1 - i\bar{\lambda}_2 D_p^\mu(0;0)][1 - i\bar{\lambda}_2 D_{p-q}^\mu(0;0)]} \right], \end{aligned} \quad (4.6)$$

plus a possible tadpole term. Since the remaining integration, d^4q , diverges in the ultraviolet it must be regularized, and this is most conveniently done using dimensional regularization.

Of most interest for phenomenological purposes is the contribution to the oblique parameters S , T and U [1, 25], which involve those terms in $\Pi_{ab}^{\mu\nu}$ having the tensor structure $(p^2\eta^{\mu\nu} - p^\mu p^\nu)$. Since the Higgs is an $SU_L(2) \times U_Y(1)$ doublet, it automatically preserves the accidental custodial $SU_c(2)$ symmetry [1, 26] that preserves the successful mass relation $M_W = M_Z \cos \theta_W$, thereby suppressing its contribution to T and making S of most interest. Because all mass dependence in eq. (4.6) is logarithmic, recalling the definition $\bar{\kappa}_b = 1/f^2$ and extracting the conventional factors of $g^2/4\pi$, we obtain the estimate

$$S \sim \frac{1}{4\pi\bar{\lambda}_2^2} \left(\frac{p^4}{f^4} \right), \tag{4.7}$$

where p^2 represents the momentum transfer of interest. Applied to LEP experiments we may take $p^4 = M_Z^4$ and $|S| < 0.1$ to conclude $\bar{\lambda}_2 f^2 \gtrsim v^2$.

4.2 Higgs decays to fermions

Another class of observables involve specifying a specific Higgs KK mode. Perhaps the simplest of these is the decay rate for specific Higgs states into SM particles (although this decay need not dominate the lifetime of a given KK mode because it must also compete with other channels, such as off-brane decays into the Goldstone modes $\delta\zeta_i$).

Generic KK states. For simplicity we start with the decay of a generic KK mode into brane fermions, assuming the KK wave-functions, $\Psi(y)$, extend throughout much of the extra-dimensional bulk so that $|\Psi(0)|^2 \simeq 1/V_2 \simeq M_c^2$. Once excited, such a heavy state can decay through the interaction (3.12), with the rate

$$\Gamma(\chi \rightarrow \bar{\psi}_i \psi_j) \simeq |\Psi(0)|^2 \frac{|y_{ij}|^2}{F^2} M_\chi \simeq |y_{ij}|^2 \left(\frac{M_c}{F} \right)^2 M_\chi, \tag{4.8}$$

where $M_\chi \geq m_B$ is the mass of the decaying mode. (Recall that the bulk Goldstone modes, $\delta\zeta_i$, do not decay in this way because of the requirement that they vanish at the brane.) We see that $\Gamma \ll M_\chi$ naturally follows from the smallness of the quantities y_{ij} and M_c/F (the latter of which is particularly small in the case of large extra dimensions). Whether these are the dominant decay channels depends on the availability of light states in the bulk (or on other branes) into which competing decays can proceed, and how efficiently these Higgs decays occur.

The self-localized state. Notice that y_{ij} vanishes, strictly speaking, when $\epsilon \rightarrow 0$ with \bar{y}_{ij} and $\bar{\lambda}_2$ held fixed (making eq. (4.8) vanish logarithmically in this limit). The same is not true of the self-localized state, whose wave-function also diverges logarithmically at the position of the brane as $\epsilon \rightarrow 0$. In this case the decay rate can be computed using the interaction of eq. (3.16), leading (on neglect of final-state fermion masses) to the standard

4D expression

$$\Gamma(h \rightarrow \bar{\psi}_i \psi_j) = \frac{1}{8\pi} |\hat{y}_{ij}|^2 m_h = \Gamma^{\text{sm}}(h \rightarrow \bar{\psi}_i \psi_j) \left(\frac{m_B^2}{\pi f^2}\right) \left(\frac{2\pi}{\lambda_{2\star}}\right)^2 e^{-16\pi/|\lambda_{2\star}|}, \quad (4.9)$$

which remains nonzero as $\epsilon \rightarrow 0$. This drops dramatically, as required in the unlocalized limit, as $|\lambda_{2\star}| \rightarrow 0$, and scales as $\Gamma^{\text{sm}} m_h^4 / (16\pi m_B^2 f^2)$ when $|\lambda_{2\star}| \gg 2\pi$.

4.3 TeV-scale compactifications

Suppose, first, the compactification scale, M_c , lies in the TeV range and so, in the absence of significant warping, the 4D Planck scale, $M_p \sim M_*^2/M_c$, comes out right if $M_* \sim 10^{10}$ GeV. This leaves lots of room to choose the other scales of interest to be much smaller than M_* in order to justify our neglect of gravitational interactions. We do not speculate as to how the extra-dimensional size is stabilized at this scale.

Choosing M_c this large also ensures that this is the mass of the lightest KK mode of the bulk Goldstone bosons, $\delta\zeta_i$, ensuring that these modes do not play much of a phenomenological role until energies are reached — at the LHC, God willing — that allow the direct production of KK excitations. The same is true of the generic KK modes of the field χ , provided we also choose m_B to be large enough.

We have seen that the absence of Higgs detection in oblique or in 2-fermion to 2-fermion processes implies us to choose $f\sqrt{|\lambda_{2\star}|}$ to be at least several hundred GeV, whereas our use of a 6D calculational framework requires both f and $M_b \sim \sqrt{vf}(2\pi/|\lambda_{2\star}|)^{1/4}$ to be $\gtrsim M_c$. There are then two subcategories to consider, depending on the size of $|\lambda_{2\star}|/2\pi$.

Weak localization. Consider first the limit of small $|\lambda_{2\star}|$, for which $m_h \rightarrow m_B$ and $k \rightarrow 0$. Because k is small, the ‘bound’ state is not strongly localized relative to generic extra dimensional scales, and the breakdown of the approximation $k \gg M_c$ demands we go beyond the simple large-volume limits used above for the scalar v.e.v. and wave-function. Taking $|\lambda_{2\star}| \sim 0.01$ for illustrative purposes, we see that requiring $f > M_c \sim \text{few TeV}$ automatically ensures $f\sqrt{|\lambda_{2\star}|} \gtrsim$ several hundred GeV, and so is large enough to avoid the phenomenological bounds.

For weak localization, the exponential suppression of \hat{y}_{ij} for small $|\lambda_{2\star}|$ allows us to choose m_B to be much larger than f without the Higgs-fermion couplings becoming strong. However we cannot have all χ states be too much higher than the TeV scale without there being a breakdown of the low-energy effective theory, such as through the development of unitarity problems in the scattering of longitudinal W bosons that the SM would suffer in the absence of a low-energy Higgs particle [1, 27, 28], and this puts an upper bound on how large m_B can be. In this case the χ spectrum resembles the usual intuition for bulk fields in the absence of brane couplings, consisting of a tower of Higgs KK modes starting above the gap at m_B . Furthermore, because these particles are likely to have a significant decay rate into the lighter bulk Goldstone states, any observed Higgs is likely to have a significant invisible width.

Because m_B cannot be made exceedingly large without running into troubles, and because M_c is typically smaller, it should be possible to observe some of the Higgs KK

states at the LHC. Although the mass- M_c Goldstone states cost less energy, they are more difficult to produce because of the absence of direct couplings to the initial brane-based SM particles. The most likely channel for doing so is the virtual excitation of KK modes of the bulk state χ . Convincing evidence for these Goldstone states together with an absence for KK modes for the electroweak gauge bosons would provide the smoking gun for this scenario: with the Higgs in the bulk but gauge interactions localized to live only on the branes.

Strong localization. In the opposite limit, $|\lambda_{2\star}| \gg 2\pi$, the lowest energy state becomes localized to the brane with $k \simeq m_B$, and its mass drops to $m_h^2 \simeq 8\pi m_B^2 / |\lambda_{2\star}| \ll m_B^2$. In this case m_B can be higher than it could for weak localization, provided that the self-localized state is lighter than a few TeV and so can unitarize the scattering of longitudinal gauge boson modes.

An upper limit to how large m_B can be is found from the condition that this light, localized Higgs state be weakly coupled

$$\left| \frac{\hat{y}_{ij}}{y_{ij}^{\text{SM}}} \right|^2 \simeq \frac{8}{\pi} \left(\frac{m_B}{f} \right)^2 \left| \frac{2\pi}{\lambda_{2\star}} \right|^3 \simeq \frac{1}{8\pi} \left(\frac{m_h}{f} \right)^2 \left(\frac{m_h}{m_B} \right)^4. \quad (4.10)$$

Large $|\lambda_{2\star}|$ also implies that the condition $f > M_c$ automatically ensures the validity of the phenomenological limits that require $f\sqrt{|\lambda_{2\star}|}$ to be larger than several hundred GeV, and makes the strongest constraint on f the theoretical condition that M_b be larger than M_c .

For instance for moderately large $|\lambda_{2\star}|/2\pi \sim 10^2$, then keeping m_h at the TeV scale requires $m_B \simeq 10$ TeV, and taking $M_b \sim 1$ TeV then implies $f \sim 10$ TeV. By contrast, if m_B should be the largest scale considered so far, $m_B \sim M_* \sim 10^{10}$ GeV, then $|\lambda_{2\star}|/2\pi \sim 10^{20}$, and so $M_b \sim 10^{-5}f > 1$ TeV implies a strong hierarchy between M_b and $f > 10^5$ GeV whose naturalness would have to be explained. Notice that the physical couplings, \hat{y}_{ij} , are much smaller than for the SM given these scales.

In this case m_B could easily be large enough to preclude the direct detection of a Higgs KK spectrum, even at the LHC, leaving the burden of Higgs physics carried by the single self-localized Higgs state. In principle this can be distinguished from a SM Higgs in several ways. First, it could well have a large invisible width, if the mass of the self-localized state is sufficiently large compared with the mass, M_c , of the bulk Goldstone modes. Second, it can be distinguished by identifying the difference in the strength of its couplings to fermions from those expected in the SM.

4.4 Large extra dimensions

An alternative choice [2, 15, 29] would put the scale of extra-dimensional gravity at $M_* \sim 10$ TeV, which then requires $M_c \sim 10^{-2}$ eV. As a result, the upper bound $m_B < M_*$ automatically keeps the generic Higgs KK modes light enough to potentially be seen at the LHC, yet absence of the detection of Higgs KK modes also implies m_B cannot be much below the TeV scale.

An automatic consequence of having M_c so small is to make the bulk Goldstone states, $\delta\zeta_i$, essentially massless. This ensures that they are always kinematically available as final states for χ decays, making a significant invisible width for this state inevitable. In fact, the very lightest KK Goldstone modes in this scenario are light enough to mediate forces between macroscopic bodies, with generically near-gravitational strength, making them potentially relevant to precision tests of Newton's inverse-square law for gravity. Their presence is nonetheless unlikely to have been already ruled out due to the absence of direct couplings to brane matter, and the derivative nature of their Goldstone interactions.

In this scenario the conditions $f, M_b \gtrsim M_c$ pose no significant constraint, with more information coming from the phenomenological conditions that $f\sqrt{|\lambda_{2*}|}$ be larger than a few hundred GeV. Notice that if we also require $f \lesssim M_*$ then we must have an upper bound $|\lambda_{2*}| \lesssim 10^4$, and so the self-localized state cannot be more than a few orders of magnitude lighter than m_B .

Because the KK tower of modes is so narrowly spaced — by $O(M_c)$ — they provide almost a continuum of states. Although each of these modes couples with gravitational strength, their phase space makes their inclusive production cross section of order the weak-interaction size [2]. Once the Higgs is produced, its phenomenology is likely to resemble that of extra-dimensional gravitons [30] or other bulk matter fields [31], including likely large invisible decay channels.

5. Conclusions

In this paper we examine a new way for brane-world scenarios to change how we think about low-energy naturalness problems. We do so by showing how oft-neglected couplings to branes can dramatically change the vacuum energetics and low-energy spectrum for bulk scalar fields. In particular, we show that when coupled to codimension-2 branes bulk scalar fields can have two unusual properties:

- They can acquire v.e.v.s that are only logarithmically related to the size of the UV-sensitive quadratic term, $\frac{1}{2}m_B^2\phi^2$, in the bulk Higgs potential;
- They can acquire low-energy KK modes that are localized to the branes (without the need for warping), and whose mass can lie inside the naive gap below the energy set by the mass scale m_B .

We further use these two observations to explore the possibility of building phenomenological brane-world models for which all Standard Model particles (save the Higgs) are trapped on a brane, but with the Higgs allowed to live in the bulk. We estimate the size of the effective couplings of such a Higgs to gauge bosons and fermions on the brane, and use these to estimate the sizes of masses and couplings to the Higgs KK modes.

We do not try to identify ultraviolet completions of the bulk-Higgs model, and so do not identify at a microscopic level why the electroweak hierarchy exists in the first place. Our focus is instead on whether such a hierarchy can be technically natural purely within the low-energy theory. We identify in eq. (3.3) the main obstacle to systematically raising

the UV scale of this effective theory above the weak scale, since this equation generically requires the two dimensionful parameters f and M_b — governing the size of the brane potential term $(H^*H)^2/M_b^4$ and the brane kinetic term $(D_M H^* D^M H)/f^2$ — either to satisfy $M_b \sim f\sqrt{2\pi/\lambda_{2^*}}$ with both near the electroweak scale, or to satisfy the hierarchy $M_b \ll f\sqrt{2\pi/\lambda_{2^*}}$ if both are large compared with the electroweak scale. This latter hierarchy shows how the problem gets recast with a bulk Higgs, since both interactions are allowed by the same symmetries, making it unnatural for them to have coefficients suppressed by very different scales.

We provide a very preliminary discussion of possible signals and constraints on these models, including the observation that most realizations predict a significant invisible width for any observed ‘Higgs’, once detected. Simple estimates are made of Higgs decay rates into SM particles, the scattering rate for fermions due to virtual Higgs exchange, and the contribution of virtual Higgs loops to gauge boson vacuum polarization. These are used to outline the qualitative features of Higgs phenomenology within this class of models. In all cases we find that the phenomenology of these models is sufficiently interesting to bear further, more detailed study.

Acknowledgments

We wish to thank Georges Azuelos, Massimo Giovannini, Quim Matias and James Wells for helpful comments and suggestions. This research has been supported in part by funds from the Natural Sciences and Engineering Research Council (NSERC) of Canada. CB also acknowledges support from CERN, the Killam Foundation, and McMaster University. CdR is funded by an Ontario Ministry of Research and Information (MRI) postdoctoral fellowship. Research at the Perimeter Institute is supported in part by the Government of Canada through NSERC and by the Province of Ontario through MRI.

A. Some properties of Bessel functions

This appendix summarizes a few properties of modified Bessel functions which are used in the main text. The modified Bessel functions are linearly independent solutions to the differential equation

$$z^2 y'' + zy' - (z^2 + \nu^2)y = 0, \tag{A.1}$$

with $I_\nu(z)$ chosen to be regular at $z = 0$ and $K_\nu(z)$ chosen to fall off to zero as $z \rightarrow \infty$. They are defined in terms of ordinary Bessel functions, $J_\nu(z)$, and Hankel functions, $H_\nu^{(1)}(z)$, by

$$I_\nu(z) = i^{-\nu} J_\nu(iz) \quad \text{and} \quad K_\nu(z) = \frac{\pi}{2} i^{\nu+1} H_\nu^{(1)}(iz). \tag{A.2}$$

The expansion of these functions for small argument is used in the text. For $0 < z \ll \sqrt{\nu+1}$ it is given by

$$I_\nu(z) \simeq \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu, \quad K_0(z) \simeq -\ln\left(\frac{z}{2}\right) - \gamma \quad \text{and} \quad K_\nu(z) \simeq \frac{\Gamma(\nu)}{2} \left(\frac{2}{z}\right)^\nu \quad \text{if } \nu > 0. \tag{A.3}$$

The asymptotic form at large z is similarly given (for $z \gg |\nu^2 - \frac{1}{4}|$) by

$$I_\nu(z) \simeq \frac{1}{\sqrt{2\pi z}} e^z \quad \text{and} \quad K_\nu(z) \simeq \sqrt{\frac{\pi}{2z}} e^{-z}. \quad (\text{A.4})$$

The energy integral encountered in the main text can be evaluated explicitly, using the following Bessel-function identities

$$K'_\nu = -K_{\nu-1} - \frac{\nu K_\nu}{z} = -K_{\nu+1} + \frac{\nu K_\nu}{z}, \quad (\text{A.5})$$

which imply in particular $K'_0 = -K_1$, $K'_1 = -K_0 - K_1/z = -K_2 + K_1/z$ and $K'_2 = -K_1 - 2K_2/z$. Repeated application of these shows that

$$\frac{d}{dz} \left[\frac{1}{2} z^2 (K_0^2 - K_1^2) \right] = z K_0^2 \quad \text{and} \quad \frac{d}{dz} \left[\frac{1}{2} z^2 (K_1^2 - K_0 K_2) \right] = z K_1^2, \quad (\text{A.6})$$

and so

$$z (K_0^2 + K_1^2) = \frac{d}{dz} \left[\frac{1}{2} z^2 K_0 (K_0 - K_2) \right]. \quad (\text{A.7})$$

B. Classical divergences in brane couplings

This appendix summarizes the derivation of the renormalization of the codimension-2 couplings encountered in the text, with an emphasis on identifying its domain of validity.

Consider to this end the following bulk-brane quadratic action for a single real scalar field,

$$S = -\frac{1}{2} \int d^4x d^2y \left[\partial_M \phi \partial^M \phi + m_B^2 \phi^2 \right] + \frac{1}{2} \int d^4x \lambda_2 \phi^2. \quad (\text{B.1})$$

(The unusual sign for the brane term is chosen to be consistent with its use in the main text.) The exact propagator, $G(x, y; x', y')$, for this theory satisfies the differential equation

$$\left[\partial_M \partial^M - m_B^2 + \lambda_2 \delta^2(y) \right] G(x, y; x', y') = i \delta^4(x - x') \delta^2(y - y'), \quad (\text{B.2})$$

while the propagator in the absence of the brane coupling, $D(x, y; x', y')$, instead satisfies

$$\left[\partial_M \partial^M - m_B^2 \right] D(x, y; x', y') = i \delta^4(x - x') \delta^2(y - y'). \quad (\text{B.3})$$

It is useful to regard these as the position-basis representation of two abstract operators, G and D , so that $G(x, y; x', y') = \langle x, y | G | x', y' \rangle$ (and similarly for D). In this case the above relations can be written $G^{-1} = D^{-1} - iV$, where $\langle x, y | V | x', y' \rangle = \lambda_2 \delta^2(y) \delta^4(x - x') \delta^2(y - y')$. Multiplying on the left by D and on the right by G then allows this to be written as $G = D + iDVG$, whose position-basis expression is equivalent to the integral equation

$$G(x, y; x', y') = D(x, y; x', y') + i\lambda_2 \int d^4\hat{x} D(x, y; \hat{x}, 0) G(\hat{x}, 0; x', y'). \quad (\text{B.4})$$

After Fourier transforming the translation-invariant x^μ directions

$$G(x, y; x', y') = \int \frac{d^4 p}{(2\pi)^4} G_p(y; y') e^{ip \cdot (x-x')}, \quad (\text{B.5})$$

eq. (B.4) becomes the exact statement

$$G_p(y; y') = D_p(y; y') + i\lambda_2 D_p(y; 0)G_p(0; y'). \quad (\text{B.6})$$

Since this no longer involves convolutions it may be solved explicitly. Specializing first to $y = 0$ implies $G_p(0; y') = D_p(0; y')/[1 - i\lambda_2 D_p(0; 0)]$, which when re-substituted into eq. (B.6) gives

$$G_p(y; y') = D_p(y; y') + i\lambda_2 \frac{D_p(y; 0)D_p(0; y')}{1 - i\lambda_2 D_p(0; 0)}. \quad (\text{B.7})$$

Notice that no approximations have been made that implicitly restrict us to small λ_2 .

The problem with the solution, eq. (B.7), is that the quantity $D_p(0; 0)$ diverges, and this observation lies at the root of the need for renormalization. The expression for $D_p(y; y')$ may be explicitly constructed as the following mode sum, using polar coordinates $\{y^m\} = \{r, \theta\}$ in the transverse dimensions, with $r = 0$ representing the brane position:

$$D_p(r, \theta; r', \theta') = -i \sum_{n=-\infty}^{\infty} e^{in(\theta-\theta')} \int_0^\infty \left(\frac{qdq}{2\pi}\right) \frac{1}{p^2 + q^2 + m_B^2} J_n(qr) J_n(qr'), \quad (\text{B.8})$$

where⁶ $p^2 = p_\mu p^\mu$. To isolate the divergence in $D_p(0; 0)$ evaluate at $r = r' = 0$ and use $J_n(0) = \delta_{n0}$ to get

$$D_p^\Lambda(0; 0) = -i \int_0^\Lambda \left(\frac{qdq}{2\pi}\right) \frac{1}{p^2 + q^2 + m_B^2} = -\frac{i}{2\pi} \ln\left(\frac{\Lambda}{P}\right) + O\left(\frac{P^2}{\Lambda^2}\right), \quad (\text{B.9})$$

where $P^2 = p^2 + m_B^2$.

Renormalization may also be performed without resorting to an expansion in powers of λ_2 . The goal is to redefine $\lambda_2 = \bar{\lambda}_2(\Lambda) \rightarrow \bar{\lambda}_2(\mu)$ in such a way as to absorb the divergence in $D^\Lambda(0; 0)$:

$$\frac{\lambda_2}{1 - i\lambda_2 D_p^\Lambda(0; 0)} \equiv \frac{\bar{\lambda}_2(\mu)}{1 - i\bar{\lambda}_2(\mu) D_p^\mu(0; 0)}, \quad (\text{B.10})$$

or, equivalently

$$\frac{1}{\bar{\lambda}_2(\Lambda)} \equiv \frac{1}{\bar{\lambda}_2(\mu)} + i \left[D_p^\Lambda(0; 0) - D_p^\mu(0; 0) \right] = \frac{1}{\bar{\lambda}_2(\mu)} + \frac{1}{2\pi} \ln\left(\frac{\Lambda}{\mu}\right), \quad (\text{B.11})$$

in agreement with the usage in the main text.

⁶The generalization of this expression to the case where the transverse geometry has a conical defect at the brane position is given in ref. [18].

C. Higher codimension

In this appendix we examine how the arguments of §2 change for a Higgs living in a $(4 + n)$ -dimensional bulk coupled to a codimension- n brane, with $n \geq 3$.

We divide the discussion into a derivation of how the brane couplings renormalize in arbitrary codimension, and then examine the energy density that governs the size of the resulting scalar expectation value.

C.1 Coupling renormalization

We start with a discussion of brane coupling renormalization. The main complication in the higher-codimension case is the appearance of power-law divergences, with all of the pitfalls and complications which these entail for the low-energy description [28].

Consider the $(n + 4)$ -dimensional scalar field

$$S = - \int d^4x d^n y \left[\frac{1}{2} (\partial_M \phi \partial^M \phi) + \frac{1}{2} m_B^2 \phi^2 + \delta^n(y) V_b(\phi) \right], \quad (C.1)$$

with brane potential

$$V_b = -\frac{1}{2} \lambda_2 \phi^2 + \frac{1}{4} \lambda_4 \phi^4, \quad (C.2)$$

living in a flat space-time with metric

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + r^2 \gamma_{ab}(\theta) d\theta^a d\theta^b. \quad (C.3)$$

Here the θ^a are coordinates for the $n-1$ angular directions, whose total volume we denote by $\varpi = \int d^{(n-1)}\theta \sqrt{\gamma}$. We focus for simplicity on spherically symmetric solutions (independent of the angular directions), although this assumption is not crucial (since higher modes in the angular directions are regular at $r = 0$).

As for codimension 2, the relation between the propagator, G , in the presence of the brane coupling, and the propagator, D , in its absence, is

$$G_k(y; y') = D_k(y; y') + i\lambda_2 \frac{D_k(y; 0) D_k(0; y')}{1 - i\lambda_2 D_k(0; 0)}, \quad (C.4)$$

and as before the need for renormalization may be traced to the divergence in $D_k(0; 0)$. The nature of this divergence can be divined from the mode sum giving the propagator, D , in the absence of brane couplings

$$[\square - m_B^2] D(x, y; x', y') = i\delta^4(x - x') \delta^n(y - y'), \quad (C.5)$$

which, in brane-Fourier space,

$$D(x, y; x', y') = \int \frac{d^4p}{(2\pi)^4} D_p(y; y') e^{ip \cdot (x - x')}, \quad (C.6)$$

has as solution

$$D_p(x; x') = -i \int_0^\infty \frac{q^{n-1} dq}{\varpi} \frac{1}{p^2 + m_B^2 + q^2} \left[\frac{1}{(qr)^\nu} J_\nu(qr) \right] \left[\frac{1}{(qr')^\nu} J_\nu(qr') \right] + \dots, \quad (C.7)$$

with $\nu = (n - 2)/2$. The ellipses in this last equation represent those terms involving the nontrivial angular modes.

Using the asymptotic form for J_ν in the limit $qr \ll 1$: $J_\nu(qr) = (qr)^\nu / [\nu! 2^\nu] + \mathcal{O}(qr)$, we find

$$D_p(r = 0; r' = 0) = -\frac{i}{(\nu!)^2 2^{2\nu}} \int_0^\infty \frac{q^{n-1} dq}{\varpi} \frac{1}{m_B^2 + p^2 + q^2}, \quad (\text{C.8})$$

which diverges as a power of the UV cutoff, Λ , as

$$D_p^{\tilde{\Lambda}}(0; 0) = -\frac{i}{(\nu!)^2 2^{2\nu}} \int_0^{\tilde{\Lambda}} \frac{q^{n-1} dq}{\varpi} \frac{1}{m_B^2 + p^2 + q^2}. \quad (\text{C.9})$$

$$= -\frac{i}{\varpi (\nu!)^2 2^{2\nu}} \left[\frac{q^n}{nP^2} - \frac{q^{n+2}}{(n+2)P^4} {}_2F_1\left(1, \frac{n+2}{2}; \frac{n+4}{2}, -\frac{q^2}{P^2}\right) \right]_0^{\tilde{\Lambda}}, \quad (\text{C.10})$$

where $P^2 = m_B^2 + p^2$ and ${}_2F_1(a, b; c; z)$ denotes the Hypergeometric function.

Our focus is on even n , $n = 2m$, in which case the hypergeometric function can be simplified to the following terminating series

$${}_2F_1(1, m+1, m+2, z) = -(m+1)z^{-(m+1)} \left[\log(1-z) + \sum_{j=1}^m \frac{z^j}{j} \right]. \quad (\text{C.11})$$

Using this in the expression of the brane-brane propagator for even codimensions, we get

$$D_p^{\tilde{\Lambda}}(0; 0) = \frac{i2^{2-m}}{\varpi[\Gamma(m)]^2} \left[\frac{(-)^m}{2} P^{2(m-1)} \log\left(1 + \frac{q^2}{P^2}\right) + \frac{1}{2} \sum_{j=1}^{m-1} \frac{(-)^{j-m}}{j} q^{2j} P^{2(m-1-j)} \right]_0^{\tilde{\Lambda}}. \quad (\text{C.12})$$

For even codimension, $n = 2m$, we redefine $\Lambda^2 = \tilde{\Lambda}^2 + P^2$, leading to

$$D_p^\Lambda = -\frac{i2^{2-m}}{\varpi[\Gamma(m)]^2} (-P^2)^{m-1} \left[\log \Lambda + \sum_{j=1}^{m-1} \frac{1}{2j} \left(1 - \frac{\Lambda^2}{P^2}\right)^j \right] + (\text{finite}). \quad (\text{C.13})$$

For odd codimensions, a similar argument gives

$$D_p^\Lambda = -\frac{i2^{2-n}}{\varpi[\Gamma(n/2)]^2} \sum_{j=0}^{[n/2-1]} (-)^j \frac{P^{2j} \Lambda^{n-2-2j}}{n-2-2j} + (\text{finite}), \quad (\text{C.14})$$

where $[n/2 - 1]$ denotes the largest integer smaller than $n/2 - 1$.

Renormalization proceeds as for codimension two, with the requirement that

$$\frac{\lambda_2(\Lambda)}{1 - i\lambda_2(\Lambda)D_k^\Lambda(0, 0)} = \frac{\lambda_2(\mu)}{1 - i\lambda_2(\mu)D_k^\mu(0, 0)}, \quad (\text{C.15})$$

where μ is the renormalization scale, leading to the following expression,

$$\frac{1}{\lambda_2(\Lambda)} = \frac{1}{\lambda_2(\mu)} + i(D_k^\Lambda - D_k^\mu). \quad (\text{C.16})$$

The divergence of propagator on the brane also induces divergences in the expression of the 4-point function, which should be absorbed by a renormalization of λ_4 ,

$$G_{k_1, k_2, k_3, k_4}^{(4)}(y_1; y_2; y_3; y_4) = -6i \lambda_4 \left[\prod_{i=1}^4 G_{k_i}^{(2)}(y_i; 0) \right] \delta^4 \left(\sum_i k_i \right) \quad (\text{C.17})$$

$$= -6i \lambda_4 \left[\prod_{i=1}^4 \frac{D_{k_i}(y_i; 0)}{1 - i\lambda_2 D_{k_i}(0; 0)} \right] \delta^4 \left(\sum_i k_i \right). \quad (\text{C.18})$$

The quantity $\lambda_4/(1 - i\lambda_2 D_{k_i}(0, 0))^4$ is finite if λ_4 is renormalized in the following way

$$\lambda_4(\Lambda) = \frac{\bar{\lambda}_4}{(1 + i\bar{\lambda}_2 (D_k^\Lambda - D_k^\mu))^4}. \quad (\text{C.19})$$

Similar expressions can be found for higher-point couplings.

C.2 Boundary condition and energy density

We now turn to the classical solutions for $\phi(r)$, and the boundary conditions which communicate the information of the brane potential to the bulk theory. Just as in the main text the singular form of the bulk solutions require us to regularize the boundary condition by evaluating it at $r = \epsilon$ rather than at $r = 0$. Smooth results are obtained as $\epsilon \rightarrow 0$ once the bare couplings are eliminated in terms of the renormalized couplings.

The classical solution to the bulk field equation that vanishes far from the brane is

$$\phi(r) = \bar{\phi} \frac{K_\nu(m_B r)}{(m_B r)^\nu}. \quad (\text{C.20})$$

Integrating the equation of motion over the brane, we obtain the boundary condition

$$\varpi \epsilon^{n-1} \phi'_\epsilon = -\lambda_2 \phi_\epsilon + \lambda_4 \phi_\epsilon^3. \quad (\text{C.21})$$

The energy density for such a field configuration is similarly given by

$$\begin{aligned} \mathcal{H} &= v \int_\epsilon^\infty r^{n-1} dr \left[\frac{1}{2} (\partial_r \phi)^2 + \frac{1}{2} m_B^2 \phi^2 \right] + U_b(\phi(\epsilon)) \\ &= v \frac{m_B^2}{2} \bar{\phi}^2 \epsilon^{n+1} (m_B \epsilon)^{-n} K_\nu(m_B \epsilon) K_{\nu+1}(m_B \epsilon) + U_b(\phi(\epsilon)). \end{aligned} \quad (\text{C.22})$$

In general both of these last equations become finite once expressed in terms of renormalized quantities, although the cancellation becomes more regularization dependent in the higher-codimension case due to the appearance there of power-law divergences rather than logarithms. Rather than working this through in complete generality, we restrict ourselves here to an illustrative calculation for codimension three.

C.3 Codimension-3

For a codimension-3 brane the divergent part of the brane-brane propagator goes as

$$D_p^\Lambda = -\frac{2i\Lambda}{\pi\varpi}, \tag{C.23}$$

and so the divergent part of the boundary condition (C.21) cancels identically if $2\Lambda/\pi = \epsilon$. The leading order part of the boundary condition becomes

$$\bar{\phi} \left(m_B + \frac{\varpi}{\lambda_2} - \mu + \frac{\pi\varpi^3\bar{\lambda}_4}{2\bar{\lambda}_2^4 m_B^2} \bar{\phi}^2 \right) = 0, \tag{C.24}$$

where to simplify the notation we rescale $\mu \rightarrow \pi\mu/2$. The system has solution $\bar{\phi} = 0$ as well as

$$\bar{\phi}^2 = -\left(\frac{2\bar{\lambda}_2^4 m_B^2}{\pi\varpi^3 \bar{\lambda}_4} \right) m_{\text{eff}}, \tag{C.25}$$

although the second solution is only possible when

$$m_{\text{eff}} = \left(\frac{\varpi}{\lambda_2} - \mu + m_B \right) < 0. \tag{C.26}$$

These conclusions are consistent with the form of the energy density, which in this case is

$$\mathcal{H} = \left(\frac{\pi\varpi}{4m_B^2} \right) m_{\text{eff}} \bar{\phi}^2 + \frac{\bar{\lambda}_4}{4} \left(\frac{\varpi}{\bar{\lambda}_2 m_B} \right)^4 \left(\frac{\pi}{2} \right)^2 \bar{\phi}^4. \tag{C.27}$$

Notice that the criterion for having a nonzero v.e.v. in this case depends more strongly on m_B , relative to the codimension-2 case.

A similar argument can be made for higher codimensions. Notice that for codimension-4 and higher, the propagator includes sub-leading divergences which should also be renormalized. Doing so, we recover a finite energy density with slightly different criteria on having a nonzero v.e.v.

D. Bulk Goldstone modes

A natural worry arises when the Higgs is regarded as a bulk scalar while the Standard Model gauge bosons are confined to a brane. Since the bulk $SU(2) \times U(1)$ rotations are not gauged, their spontaneous breaking might be expected to bulk Goldstone modes, corresponding to KK towers of bulk scalar modes whose lightest members are massless (or with masses set by the KK scale, if the global symmetries are broken by boundary conditions). Since only three of these 4D KK states are eaten by the Higgs mechanism, the remainder could survive and generate a potentially dangerous large number of light states. In this section, we show that only three massless Goldstone modes are produced, all of which are eaten by the gauge fields on the brane.

We start with the argument in a nutshell: when choosing a specific vacuum, such as the unitary gauge choice of the main text, one expects Goldstone modes connecting to nearby vacua. Since all vacua have the same profile in the extra dimensions, the Goldstone modes

also share this profile. The modes with the smallest energy cost have only momentum along the brane directions, and so are effectively already four-dimensional. These modes turn out to be the self-localized states of those components of the Higgs doublet that do not acquire a v.e.v.

To see this explicitly we repeat the calculation of the light states in section 2.3, for the Higgs doublet H . The equation of motion analogous to eq. (2.33) is

$$\left[\frac{1}{r^2} ((r\partial_r)^2 + \partial_\theta^2) + \partial_\mu \partial^\mu - m_B^2 \right] H = \frac{\delta_+(r)}{2\pi r} (-\lambda_2 + \lambda_4 H^* H) H. \quad (D.1)$$

This equation may be linearized around the vacuum by setting

$$H = \begin{pmatrix} 0 \\ \varphi(r) \end{pmatrix} + \sum_{n \geq 0} \begin{pmatrix} \zeta_1^n(r) + i\zeta_2^n(r) \\ \chi^n(r) + i\zeta_3^n(r) \end{pmatrix} \sin(n\theta), \quad (D.2)$$

with $\varphi(r) = \bar{\phi} K_0(m_B r)$ and where we introduce an infinite tower of excitation modes along the angular direction. Each one of these modes satisfies the equations of motion

$$\left[\frac{1}{r} \partial_r (r \partial_r) - \frac{n^2}{r^2} - k^2 \right] \chi^n = \frac{\delta_+(r)}{2\pi r} (-\lambda_2 + 3\lambda_4 \varphi^2(r)) \chi^n \quad (D.3)$$

$$\left[\frac{1}{r} \partial_r (r \partial_r) - \frac{n^2}{r^2} - k^2 \right] \zeta_i^n = \frac{\delta_+(r)}{2\pi r} (-\lambda_2 + \lambda_4 \varphi^2(r)) \zeta_i^n, \quad (D.4)$$

where, as before, $k^2 = m_B^2 - \omega^2$.

The field χ^0 is the ‘physical’ self-localized state, discussed in the main text, and has a mass as calculated in eq. (2.37) with (2.38). The same goes through for the zero mode of the other fields ζ_i^0 , but taking into account the different factor of λ_4 between equations (D.3) and (D.4), their masses are given by

$$\omega_{\zeta^0}^2 = m_B^2 \left[1 - e^{-4\pi/\lambda_{\zeta^0}} \right], \quad (D.5)$$

with now

$$\frac{1}{\lambda_{\zeta^0}} = \frac{1}{\lambda_{2\star}} \left[1 + \frac{4\pi^2 \bar{\lambda}_4 \bar{\phi}^2}{\lambda_{2\star}^3} \right]. \quad (D.6)$$

In the broken phase, $\bar{\phi}$ is given by eq. (2.30) which leads to

$$\frac{1}{\lambda_{\zeta^0}} = 0, \quad (D.7)$$

showing there are three massless 4D Goldstone modes, ζ_i^0 . The bulk profile of these modes is enforced by the boundary condition imposed on the brane, and as argued in section 3.1, choosing unitary gauge on the brane removes these three massless states as they become ‘eaten’ by the brane gauge fields.

Turning now to the infinite tower of angular dependent modes ($n \neq 0$), the profile of these modes is now of the form $\chi^n, \zeta_i^n = N_i^n K_n(kr)$, where N_i^n is the normalization

constant and we expect k to be determined by the the boundary condition (2.35) which takes the form

$$2\pi r \partial_r \left(\begin{array}{c} \chi^n(r) \\ \zeta_i^n(r) \end{array} \right) \Big|_\epsilon = \left(-\lambda_2 + \binom{3}{1} \lambda_4 \varphi(r)^2 \right) \left(\begin{array}{c} \chi^n \\ \zeta_i^n \end{array} \right) \Big|_\epsilon. \quad (\text{D.8})$$

In the limit $\epsilon \rightarrow 0$, this reduces to

$$2^n \pi N_i^n (k\epsilon)^{-n} \left(n! + \frac{(n-1)!}{\log(\epsilon m_B e^\gamma/2)} \right) = \mathcal{O}((k\epsilon)^{-n+2}). \quad (\text{D.9})$$

We see we must have $N_i^n = 0$ if these modes are to remain bounded, and so there are therefore no light modes of this form having $\omega < m_B$. All the remaining excitations along the radial direction form a Kaluza-Klein tower of states starting at the bulk mass m_B and are thus harmless. There are therefore only three massless states ζ_i^0 playing the role of four-dimensional Goldstone modes, one self-localized massive mode (χ) with mass $0 < m < m_B$ and a tower of Higgs excitations with mass higher than the bulk mass.

References

- [1] For a recent summary of these issues see, C.P. Burgess and G.D. Moore, *The standard model: a primer*, Cambridge University Press, Cambridge U.K. (2007).
- [2] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [[hep-ph/9803315](#)];
Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity, *Phys. Rev.* **D 59** (1999) 086004 [[hep-ph/9807344](#)];
 I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [[hep-ph/9804398](#)].
- [3] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [[hep-ph/9905221](#)]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [[hep-th/9906064](#)].
- [4] A. Pomarol and M. Quirós, *The standard model from extra dimensions*, *Phys. Lett.* **B 438** (1998) 255 [[hep-ph/9806263](#)];
 A. Delgado, A. Pomarol and M. Quirós, *Supersymmetry and electroweak breaking from extra dimensions at the TeV-scale*, *Phys. Rev.* **D 60** (1999) 095008 [[hep-ph/9812489](#)];
 I. Antoniadis, *A Possible new dimension at a few TeV*, *Phys. Lett.* **B 246** (1990) 377;
 T. Gherghetta and A. Pomarol, *Bulk fields and supersymmetry in a slice of AdS*, *Nucl. Phys.* **B 586** (2000) 141 [[hep-ph/0003129](#)].
- [5] L.J. Hall, Y. Nomura and D. Tucker-Smith, *Gauge-Higgs unification in higher dimensions*, *Nucl. Phys.* **B 639** (2002) 307 [[hep-ph/0107331](#)];
 I. Gogoladze, Y. Mimura and S. Nandi, *Unification of gauge, Higgs and matter in extra dimensions*, *Phys. Lett.* **B 562** (2003) 307 [[hep-ph/0302176](#)].
- [6] G. Burdman and Y. Nomura, *Unification of Higgs and gauge fields in five dimensions*, *Nucl. Phys.* **B 656** (2003) 3 [[hep-ph/0210257](#)];

- N. Haba and Y. Shimizu, *Gauge-Higgs unification in the 5 dimensional E_6 , E_7 and E_8 GUTs on orbifold*, *Phys. Rev. D* **67** (2003) 095001 [*Erratum ibid.* **69** (2004) 059902] [[hep-ph/0212166](#)];
- N. Haba and T. Yamashita, *Dynamical symmetry breaking in gauge-Higgs unification of 5D, $N = 1$ SUSY theory*, *JHEP* **04** (2004) 016 [[hep-ph/0402157](#)];
- G. Martinelli, M. Salvatori, C.A. Scrucca and L. Silvestrini, *Minimal gauge-Higgs unification with a flavour symmetry*, *JHEP* **10** (2005) 037 [[hep-ph/0503179](#)];
- M. Sakamoto and K. Takenaga, *Large gauge hierarchy in gauge-Higgs unification*, *Phys. Rev. D* **75** (2007) 045015 [[hep-th/0609067](#)].
- [7] C.A. Scrucca, M. Serone, L. Silvestrini and A. Wulzer, *Gauge-Higgs unification in orbifold models*, *JHEP* **02** (2004) 049 [[hep-th/0312267](#)];
 C. Biggio and M. Quirós, *Higgs-gauge unification without tadpoles*, *Nucl. Phys. B* **703** (2004) 199 [[hep-ph/0407348](#)];
 Y. Hosotani, S. Noda and K. Takenaga, *Dynamical gauge-Higgs unification in the electroweak theory*, *Phys. Lett. B* **607** (2005) 276 [[hep-ph/0410193](#)];
 D. Hernandez, S. Rigolin and M. Salvatori, *Symmetry breaking in six dimensional flux compactification scenarios*, [arXiv:0712.1980](#).
- [8] E. Dudas, C. Papineau and V.A. Rubakov, *Flowing to four dimensions*, *JHEP* **03** (2006) 085 [[hep-th/0512276](#)].
- [9] E. Dudas and C. Papineau, *Dual realizations of dynamical symmetry breaking*, *JHEP* **11** (2006) 010 [[hep-th/0608054](#)].
- [10] F. Coradeschi, S. De Curtis, D. Dominici and J.R. Pelaez, *Modified spontaneous symmetry breaking pattern by brane- bulk interaction terms*, *JHEP* **04** (2008) 048 [[arXiv:0712.0537](#)].
- [11] J. Vinet and J.M. Cline, *Can codimension-two branes solve the cosmological constant problem?*, *Phys. Rev. D* **70** (2004) 083514 [[hep-th/0406141](#)].
- [12] M. Peloso, L. Sorbo and G. Tasinato, *Standard 4D gravity on a brane in six dimensional flux compactifications*, *Phys. Rev. D* **73** (2006) 104025 [[hep-th/0603026](#)];
 E. Papantonopoulos, A. Papazoglou and V. Zamarias, *Regularization of conical singularities in warped six-dimensional compactifications*, *JHEP* **03** (2007) 002 [[hep-th/0611311](#)];
 B. Himmetoglu and M. Peloso, *Isolated Minkowski vacua and stability analysis for an extended brane in the rugby ball*, *Nucl. Phys. B* **773** (2007) 84 [[hep-th/0612140](#)];
 N. Kaloper and D. Kiley, *Charting the landscape of modified gravity*, *JHEP* **05** (2007) 045 [[hep-th/0703190](#)];
 C.P. Burgess, D. Hoover and G. Tasinato, *UV caps and modulus stabilization for 6D gauged chiral supergravity*, *JHEP* **09** (2007) 124 [[arXiv:0705.3212](#)];
 M. Minamitsuji and D. Langlois, *Cosmological evolution of regularized branes in 6D warped flux compactifications*, *Phys. Rev. D* **76** (2007) 084031 [[arXiv:0707.1426](#)];
 F. Arroja, T. Kobayashi, K. Koyama and T. Shiromizu, *Low energy effective theory on a regularized brane in 6D gauged chiral supergravity*, *JCAP* **12** (2007) 006 [[arXiv:0710.2539](#)];
 C. Bogdanos, A. Kehagias and K. Tamvakis, *Pseudo-3-branes in a curved 6D bulk*, *Phys. Lett. B* **656** (2007) 112 [[arXiv:0709.0873](#)].
- [13] A. Vilenkin, *Gravitational field of vacuum domain walls and strings*, *Phys. Rev. D* **23** (1981) 852;
 R. Gregory and C. Santos, *Cosmic strings in dilaton gravity*, *Phys. Rev. D* **56** (1997) 1194 [[gr-qc/9701014](#)].

- [14] S.M. Carroll and M.M. Guica, *Sidestepping the cosmological constant with football-shaped extra dimensions*, [hep-th/0302067](#).
- [15] Y. Aghababaie, C.P. Burgess, S.L. Parameswaran and F. Quevedo, *Towards a naturally small cosmological constant from branes in 6D supergravity*, *Nucl. Phys. B* **680** (2004) 389 [[hep-th/0304256](#)].
- [16] G.W. Gibbons, R. Güven and C.N. Pope, *3-branes and uniqueness of the Salam-Sezgin vacuum*, *Phys. Lett. B* **595** (2004) 498 [[hep-th/0307238](#)];
 Y. Aghababaie et al., *Warped brane worlds in six dimensional supergravity*, *JHEP* **09** (2003) 037 [[hep-th/0308064](#)];
 P. Bostock, R. Gregory, I. Navarro and J. Santiago, *Einstein gravity on the codimension 2 brane?*, *Phys. Rev. Lett.* **92** (2004) 221601 [[hep-th/0311074](#)];
 C.P. Burgess, F. Quevedo, G. Tasinato and I. Zavala, *General axisymmetric solutions and self-tuning in 6D chiral gauged supergravity*, *JHEP* **11** (2004) 069 [[hep-th/0408109](#)];
 J. Vinet and J.M. Cline, *Codimension-two branes in six-dimensional supergravity and the cosmological constant problem*, *Phys. Rev. D* **71** (2005) 064011 [[hep-th/0501098](#)].
- [17] W.D. Goldberger and M.B. Wise, *Renormalization group flows for brane couplings*, *Phys. Rev. D* **65** (2002) 025011 [[hep-th/0104170](#)].
- [18] C. de Rham, *The effective field theory of codimension-two branes*, *JHEP* **01** (2008) 060 [[arXiv:0707.0884](#)]; *Classical renormalization of codimension-two brane couplings*, *AIP Conf. Proc.* **957** (2007) 309 [[arXiv:0710.4598](#)].
- [19] D. Langlois and M. Sasaki, *Massive scalar states localized on a de Sitter brane*, *Phys. Rev. D* **68** (2003) 064012 [[hep-th/0302069](#)].
- [20] W. Frank, D.J. Land and R.M. Spector, *Singular potentials*, *Rev. Mod. Phys.* **43** (1971) 36;
 G. Parisi and F. Zirilli, *Anomalous dimensions in one-dimensional quantum field theory*, *J. Math. Phys.* **14** (1973) 243;
 H.E. Camblong, L.N. Epele, H. Fanchiotti and C.A. Garcia Canal, *Renormalization of the inverse square potential*, *Phys. Rev. Lett.* **85** (2000) 1590 [[hep-th/0003014](#)];
 S.R. Beane et al., *Singular potentials and limit cycles*, *Phys. Rev. A* **64** (2001) 042103 [[quant-ph/0010073](#)].
- [21] E. Braaten and D. Phillips, *The renormalization group limit cycle for the $1/R^2$ potential*, [hep-th/0403168](#).
- [22] See for instance, C.P. Burgess, *Goldstone and pseudo-Goldstone bosons in nuclear, particle and condensed-matter physics*, *Phys. Rept.* **330** (2000) 193 [[hep-th/9808176](#)].
- [23] For a discussion of these issues in the codimension-1 case, see, A. Kehagias and K. Tamvakis, *Bulk Higgs with 4D gauge interactions*, *Phys. Lett. B* **628** (2005) 262 [[hep-th/0507130](#)].
- [24] S.L. Glashow, J. Iliopoulos and L. Maiani, *Weak Interactions with Lepton-Hadron Symmetry*, *Phys. Rev. D* **2** (1970) 1285.
- [25] M.E. Peskin and T. Takeuchi, *A new constraint on a strongly interacting Higgs sector*, *Phys. Rev. Lett.* **65** (1990) 964; *Estimation of oblique electroweak corrections*, *Phys. Rev. D* **46** (1992) 381;
 G. Altarelli and R. Barbieri, *Vacuum polarization effects of new physics on electroweak processes*, *Phys. Lett. B* **253** (1991) 161;
 G. Altarelli, R. Barbieri and S. Jadach, *Toward a model independent analysis of electroweak data*, *Nucl. Phys. B* **369** (1992) 3 [*Erratum ibid.* **376** (1992) 444];

- C.P. Burgess, S. Godfrey, H. Konig, D. London and I. Maksymyk, *A global fit to extended oblique parameters*, *Phys. Lett.* **B 326** (1994) 276 [[hep-ph/9307337](#)];
 C.P. Burgess, S. Godfrey, H. Konig, D. London and I. Maksymyk, *Model independent global constraints on new physics*, *Phys. Rev.* **D 49** (1994) 6115 [[hep-ph/9312291](#)].
- [26] S. Weinberg, *Implications of dynamical symmetry breaking: an addendum*, *Phys. Rev.* **D 19** (1979) 1277;
 L. Susskind, *Dynamics of spontaneous symmetry breaking in the Weinberg-Salam theory*, *Phys. Rev.* **D 20** (1979) 2619;
 P. Sikivie, L. Susskind, M.B. Voloshin and V.I. Zakharov, *Isospin breaking in technicolor models*, *Nucl. Phys.* **B 173** (1980) 189;
 H. Georgi and D.B. Kaplan, *Composite Higgs and custodial SU(2)*, *Phys. Lett.* **B 145** (1984) 216.
- [27] J.M. Cornwall, D.N. Levin and G. Tiktopoulos, *Derivation of gauge invariance from high-energy unitarity bounds on the S matrix*, *Phys. Rev.* **D 10** (1974) 1145;
 B.W. Lee, C. Quigg and H. Thacker, *Weak interactions at very high energies: the role of the Higgs-boson mass*, *Phys. Rev.* **D 16** (1977) 1519;
 M. Veltman, *Second threshold in weak interactions*, *Acta Phys. Polon.* **B8** (1977) 475.
- [28] C.P. Burgess and D. London, *Uses and abuses of effective Lagrangians*, *Phys. Rev.* **D 48** (1993) 4337 [[hep-ph/9203216](#)]; *On anomalous gauge boson couplings and loop calculations*, *Phys. Rev. Lett.* **69** (1992) 3428.
- [29] C.P. Burgess, J. Matias and F. Quevedo, *MSLED: a minimal supersymmetric large extra dimensions scenario*, *Nucl. Phys.* **B 706** (2005) 71 [[hep-ph/0404135](#)].
- [30] G.F. Giudice, R. Rattazzi and J.D. Wells, *Quantum gravity and extra dimensions at high-energy colliders*, *Nucl. Phys.* **B 544** (1999) 3 [[hep-ph/9811291](#)];
 T. Han, J.D. Lykken and R.-J. Zhang, *On Kaluza-Klein states from large extra dimensions*, *Phys. Rev.* **D 59** (1999) 105006 [[hep-ph/9811350](#)].
- [31] D. Atwood et al., *Supersymmetric large extra dimensions are small and/or numerous*, *Phys. Rev.* **D 63** (2001) 025007 [[hep-ph/0007178](#)];
 I. Antoniadis and K. Benakli, *Large dimensions and string physics in future colliders*, *Int. J. Mod. Phys.* **A 15** (2000) 4237 [[hep-ph/0007226](#)];
 J.L. Hewett and D. Sadri, *Supersymmetric extra dimensions: gravitino effects in selectron pair production*, *Phys. Rev.* **D 69** (2004) 015001 [[hep-ph/0204063](#)];
 P.H. Beauchemin, G. Azuelos and C.P. Burgess, *Dimensionless coupling of bulk scalars at the LHC*, *J. Phys.* **G 30** (2004) N17 [[hep-ph/0407196](#)];
 G. Azuelos, P.H. Beauchemin and C.P. Burgess, *Phenomenological constraints on extra-dimensional scalars*, *J. Phys.* **G 31** (2005) 1 [[hep-ph/0401125](#)].